Wedges and plate-bandes : mechanical theories after De la Hire

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Plate-bandes are straight masonry arches and are also called «flat arches» or «lintel arches». Ideally they have the surfaces of extrados and intrados plane and horizontal. The stones or bricks have radial joints usually converging in one center. The voussoirs have the form of wedges and are in French called *claveaux*. A plate-bande is, in fact, a lintel made of several stones and the proportions of lintels and plate-bandes are similar. The proportions of plate-bandes, or in other words the relationship between the thickness *t* and the span *s*, typically vary between 1/4-1/3 in thick plate-bandes, and are less than 1/20 in the most slender ones[1]. A ratio of circa 1/8 was common in the 18th century and follows a simple geometrical rule : the center forms an equilateral triangle with the intrados, and the plate-bande should contain an arc of a circle (fig. 1). The joints are usually planes, but in some cases they present a «rebated» or «stepped» form.

Plate-bandes exert an inclined thrust as any masonry arch. This thrust is usually very high and requires either massive buttresses, or to be built in the middle of thick walls. Master builders and architects have tried since antiquity to calculate the abutment necessary for any arch. A modern architect or engineer will measure the arch thrust in units of force, kN or tons. Traditionally, the thrust has been measured as the size of the buttresses required to resist it safely. Old structural rules, thus, addressed the design problem establishing a relationship between the span and the depth of the buttress. These were empirical rules, specific for each type of arch or structure in every epoch. Thus, the typical gothic buttress is 1/4 of the vault span, but a Renaissance or baroque barrel vault will need more than 1/3 of the span[2]. A plate-bande would require more than one half of the span ; this is precisely the rule cited by the French engineer Henri Gautier[3], who tried unsuccessfully to justify it by static reasons[4]. Plates-bandes were typically used to form the lintels of windows or doors (1-2 m, typically) ; in Antiquity they were also employed, though rarely, at the gates of city walls or in niches (circa 2 m, reaching 5,2 m)[5].

Plate-bandes may show particular problems : it is not unusual that some sliding of the voussoirs can be observed, particularly in thick plate-bandes. Stepped joints (fig. 1, left)

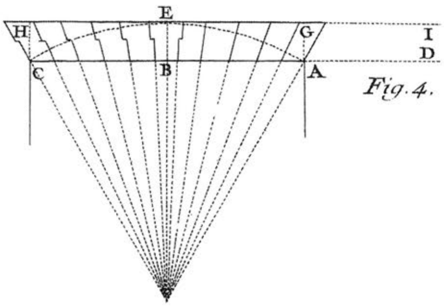
1 The plane vault in the choir of the Monastery of El Escorial (a three-dimensional plate-bande) has an span of 7,80 m with a thickness of one Castilian foot, 0,28 cm. This gives a relationship thickness/span, of circa 1/28, cf. A. López Mozo, «Planar vaults in the Monastery of El Escorial», in S. Huerta, ed., *Proceedings of the First International Congress on Construction History*, Madrid, Instituto Juan de Herrera, 2003, pp. 1327-1334. Gautier cites a plate-bande in the church of Jesuites in Nîmes with a span of four toises two feet and six inches of span (8,60 m) and a thickness of only one foot (*pied royal*, 0,325 m), giving a relationship of 1/26 (H. Gautier, *Dissertation sur l'epaisseur des culées des Ponts, sur la Largeur des piles, sur la Portée des voussoirs, sur l'Erfort & la Pesanteur des Arches à differens surbaissemens*, Paris, A. Cailleau, 1717, p. 15).

2 For a complete discussion of the traditional structural rules, see S. Huerta, *Arcos, bóvedas y cúpulas. Geometría y equilibrio en el cálculo tradicional de estructuras de fábrica*, Madrid, Instituto Juan de Herrera, 2004 (e-print in www.ad.upm.es).

3 Gautier, *op. cit.*, p. 14.

4 Huerta, *Arcos, bóvedas y cúpulas*...cit., p. 323.

5 The span of 5,2 m is in the Tabularium, with a thickness of around 1 m ; it consists of 11 voussoirs. The best contribution on ancient lintel arches is J. DeLaine, «Structural Experimentation : The Lintel Arch, Corbel and Tie in Western Roman Architecture», in *World Archaeology*, vol. XXI, 1990, pp. 407-424. See also R. Delbrueck, *Hellenistische Bauten in Latium*, Strasbourg, vol. I, 1907, pp. 24 et seqq. and *ibid.*, vol. II, 1912, p. 75.



1 A typical plate-bande. Right, with plane joints; left, with rebated or stepped joints (H. Gautier, *Dissertation sur l'epaisseur des culées des Ponts, sur la Largeur des piles, sur la Portée des voussoirs, sur l'Erfort & la Pesanteur des Arches à differens surbaissemens*, Paris, 1717, fig. 4 in pl. II).

were used to avoid this problem. Other «hidden» methods include iron cramps or the use of stone wedges, etc.; in seismic zones, these devices were usual. Another problem relates to the deformation; a slight yielding of the abutments, or even the compression of the mortar joints, may cause some cracking and the descent of the central keystone. Even a tiny descent will convert the original straight line of the intrados into a broken line with a visible «kink» or angle in the middle. Of course, both problems should be avoided. Finally, the wedge form of the voussoirs leads to acute angles in the stones and this can produce partial fractures, usually occurring at the inferior border of the springers at the abutments.

It follows that the building of a successful plate-bande is no easy matter. Also, the structural study of plate-bandes is far from simple, and mechanics and geometry are related in a particular way. The present paper concentrates on the structural aspects and their constructive consequences, with a historical approach. We outline the development of structural analysis of plate-bandes from circa 1700 until today. This brief history has a more than purely academic interest. Different approaches and theories pointed to particular problems, and though the solution given may have been incorrect, the question posed was often pertinent. The paper ends with the application of modern Limit Analysis of Masonry Structures, developed mainly by professor Jacques Heyman[6] in the last fifty years. Moreover, the work aims to give some clues for the actual architect and engineer involved in the analysis or restoration of masonry buildings.

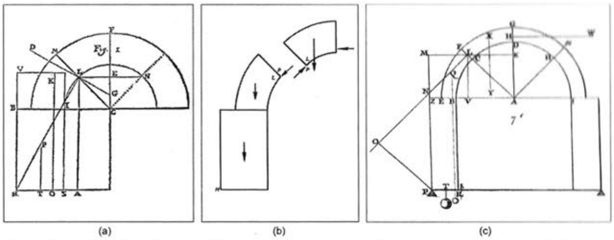
De La Hire, 1712; Bélidor, 1729

Although in the 17th century some attempts were made to understand arch and plate-bande behavior[7], it was Philippe de La Hire who first proposed a theory of arch buttresses[8]. La Hire first exposed his theory with reference to a semicircular arch: he noted that when an arch or barrel vault collapses, it breaks at some point between the keystone and the springings (the joint of rupture). He then considered that the thrust at this point should be tangent to

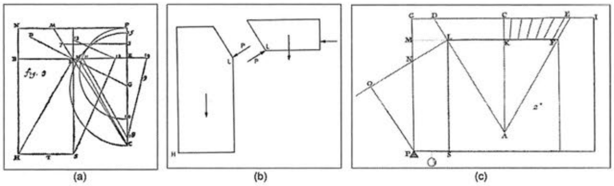
[6] The theory is exposed in J. Heyman, *The Stone Skeleton. Structural Engineering of Masonry Architecture*, Cambridge, Cambridge University Press, 1995. For a full list of author contributions see S. Huerta, ed., *Essays in the history of the theory of structures, in honour of Jacques Heyman*, Madrid, Instituto Juan de Herrera, 2005, pp. xvii-xxiv.

[7] The literature on the history of masonry arch and vault theory is quite extensive; a comprehensive bibliography (including stereotomy) is found in A. Becchi, F. Foce, *Degli Archi e delle volte. Arte del costruire tra meccanica e stereotomia*, Venice, Marsilio, 2002, pp. 251-349. The pre-eighteenth century theories have been studied by Antonio Becchi. See A. Becchi, «Before 1695: The statics of arches between France and Italy», in Huerta, ed., *Proceedings of the First International Congress on Construction History* cit., pp. 353-364; A. Becchi, *Q. XVI. Leonardo, Galileo e il caso Baldi: Magonza, 26 Marzo 1621*, Venice, Marsilio, 2005.

[8] P. de La Hire, «Sur la construction des voûtes dans les édifices», in *Mémoires de Mathématique et de Physique, in Histoire de l'Académie Royale des Sciences. Année MDCCXII. Avec les Mémoires de Mathématique et de Physique, pour la même année. Tirés des registres de cette Académie*, Paris, Imprimerie Royale, 1731, pp. 69-77.



2 Diagrams of the static of an arch. (a) P. de La Hire, drawing (P. de La Hire, «Sur la construction des voûtes dans les édifices», in *Mémoires de Mathématique et de Physique, in Histoire de l'Académie Royale des Sciences. Année MDCCXII. Avec les Mémoires de Mathématique et de Physique, pour la même année. Tirés des registres de cette Académie*, Paris, Imprimerie Royale, 1731, (pp. 69-77), p. 72). (b) J. Heyman, drawing (J. Heyman, *Structural analysis: a historical approach*, Cambridge, Cambridge University Press, 2008, p. 83). (c) B. F. de Bélidor, drawing (B. F. de Bélidor, *La science des ingénieurs dans la conduite des travaux de fortification et architecture civile*, Paris, P. Gosse, 1729, pl. IV).



3 Diagrams of the static of a plate-bande. (a) P. de La Hire, drawing (P. de La Hire, «Sur la construction des voûtes dans les édifices», in *Mémoires de Mathématique et de Physique, in Histoire de l'Académie Royale des Sciences. Année MDCCXII. Avec les Mémoires de Mathématique et de Physique, pour la même année. Tirés des registres de cette Académie*, Paris, Imprimerie Royale, 1731, (pp. 69-77), p. 72). (b) Drawing by the author. (c) B. F. Bélidor, drawing (B. F. Bélidor, *La science des ingénieurs dans la conduite des travaux de fortification et architecture civile*, Paris, P. Gosse, 1729, pl. VI).

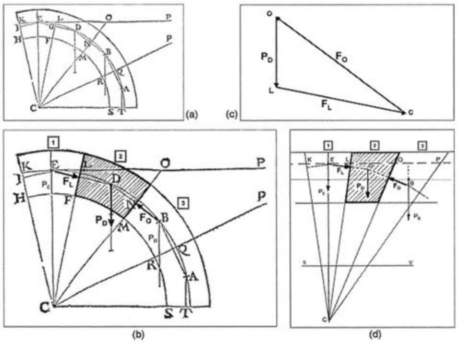
the curve of the intrados. Once the location of the joint was fixed, it is a matter of simple statics to calculate the thrust of the arch (fig. 2b). La Hire was not interested in the transmission of forces within the arch but in the design of the buttress. In the drawing, the joint of rupture is halfway (fig. 2a), but he did not state this explicitly. Besides, in establishing the equilibrium, he took moments in an awkward way, difficult to understand[9]. Bernard Forest de Bélidor simplified the method: he fixed the position of the joint of rupture at 45° from the horizontal; placed the thrust in the middle of the joint and considered it as normal to the joint. Lastly, he took moments to obtain the equilibrium in a much more simple manner.

Bélidor's aim was to develop a facile, direct, way to calculate the vault thrust. The theory was incorrect, but gave results in concordance with the proportions of the traditional empirical rules and of existing buildings. In fact, the «wrong» position and inclination of the thrust was unfavorable and contained, implicitly, a safety coefficient. It was accepted and utilized until the mid-19th century[10].

After treating the semicircular arch, La Hire passed on to the plate-bande. In this case, he considered, correctly, that the joint of rupture has to be at the joint between the horizontal intrados of the plate-bande and the inner side of the buttress (point L in figure 3a). The inclination of the springer joint renders it possible to immediately compute the thrust. La Hire took moments in the same awkward way as before, but to simplify the calculations

9 La Hire resolved the vault thrust in two directions: that of the levier HL and its normal DL. This complicates the solution. Then, he arrive correctly at a second-order equation to obtain the buttress depth. Gautier complaint about the obscurity of La Hire's deductions: «J'avoué ingenuément que je ne suis pas assez habile pour la comprendre. Je n'ay pas pû même suivre son Operation tant je la trouve composée; et je regarde tout ce qu'il nous dit, comme une chose dont le demi Sçavans, et sourtout les Ouvriers, sçauroient comprendre» (Gautier, *op. cit.*, p. 6).

10 S. Huerta, «The safety of masonry buttresses», in *Proceedings of the Institution of Civil Engineers*, Engineering history and Heritage, vol. CLXIII, 2010, pp. 3-24.



4 (a) Calculation of the weight of the voussoirs for the equilibrium without friction (P. de La Hire, *Traité de mécanique*, Paris, Imprimerie Royale, 1695). (b–c) Equilibrium of one voussoir and triangle of forces. (d) Application of De la Hire's analysis to a plate-bande (drawings by the author).

he ignored the weight of the upper part NL of the rectangular buttress. Again, Bélidor simplified the analysis, and his drawing (fig. 3c) is self-explanatory.

La Hire's theory stemmed from the classical «wedge-theory» and was technically, albeit not scientifically, correct. The architect or engineer requires a simple method to obtain valid results and La Hire, for the first time, supplied a facile straightforward means for buttress design. For example, for the plate-bande of figure 3c, where the center forms an equilateral triangle with the line of intrados, it is evident that the inclined thrust is equal to the total weight of the plate-bande[11].

Couplet, 1729

In 1729 and 1730, Pierre Couplet published a memoir on «La poussée des voûtes». It was divided into two parts. In the first part, he provided the first detailed study on vaults without friction[12]. However, his main contribution to vault theory is the second part, in which he considered sliding impossible. This second part marks the beginning of the correct theory of vaults[13]. In the first part, he considered the equilibrium of a plate-bande made of voussoirs without friction, and, therefore, for the purposes of the present article, we will concentrate on it.

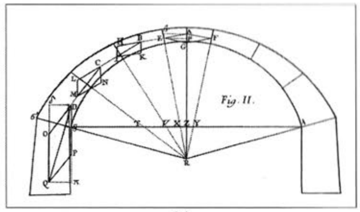
To understand Couplet and the rest of the analysis of plate-bandes without friction, we must first briefly review La Hire's analysis of an arch made of voussoirs with joints «infiniment polies», i.e., without friction[14]. La Hire was interested in calculating the weights of the voussoirs so that the whole arch would remain stable. This leads to the absurd

[11] This is the rule given by Pierre Patte : «L'action de la poussée d'une plate-bande peut être considérée toujours comme se confondant avec celle de sa pesanteur ; et sans erreur sensible, on peut apprécier l'une à l'égale de l'autre» (P. Patte, *Mémoires sur les objects les plus importants de l'architecture*, Paris, Rozet, 1769, p. 308).

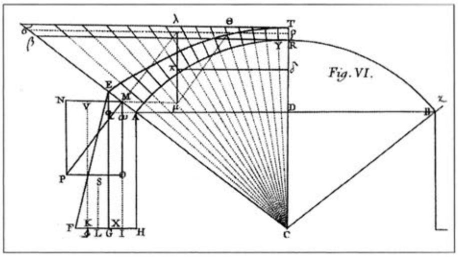
[12] P. Couplet, «De la poussée des voûtes», in *Mémoires de Mathématique et de Physique, in Histoire de l'Académie Royale des Sciences. Année MDCCXXIX. Avec les Mémoires de Mathématique et de Physique, pour la même année. Tirés des registres de cette Académie*, Paris, Imprimerie Royale, 1731, pp. 79-117, pl. IV-VII.

[13] Couplet, «Seconde partie de l'examen de la poussée des voûtes», in *Mémoires de Mathématique et de Physique, in Histoire de l'Académie Royale des Sciences. Année MDCCXXX. Avec les Mémoires de Mathématique et de Physique, pour la même année. Tirés des registres de cette Académie*, Paris, Durand, 1732, pp. 117-141, pl. VI-VII. For the importance of the work of Couplet in vault theory see, for example, J. Heyman, *The Masonry Arch*, Chichester, Ellis Horwood, 1982, pp. 50-55. On Couplet's contribution to engineering science see J. Heyman, «Couplet's Engineering Memoirs, 1726-1733», in *History of Technology*, vol. I, 1976, pp. 21-44.

[14] P. de La Hire, *Traité de méchanique*, Paris, J. Anisson, 1695, pp. 315-318. Parent studied also the problem in 1704 and he devised a simple geometrical method to obtain the curve of extrados ; he also calculated the thrust of the corresponding arch. However, it appears that the mémoire was not published and we have only the notice in the *Histoire* (A. Parent. «Sur la figure de l'extrados d'une voûte circulaire, dont tous les voussoirs sont en équilibre entre eux», in *Histoire de l'Académie Royale des Sciences, in Histoire de l'Académie Royale des Sciences. Année MDCCIV. Avec les Mémoires de Mathématique & de Physique, pour la même Année*, Paris, 1704, pp. 93-96).



(a)



(b)

5 (a) Line of thrust (trajectory of internal forces) in an arch of voussoirs without friction. Note that the form is not necessarily the same as that of the intrados or the center line. (b) Calculation of the total weight of an arch with reference to a plate-bande (P. Couplet, «De la poussée des voûtes», in *Mémoires de Mathématique et de Physique, in Histoire de l'Académie Royale des Sciences. Année MDCCXXIX. Avec les Mémoires de Mathématique et de Physique, pour la même année. Tirés des registres de cette Académie*, Paris, Imprimerie Royale, 1731, (pp. 79-117), fig. 2, 5).

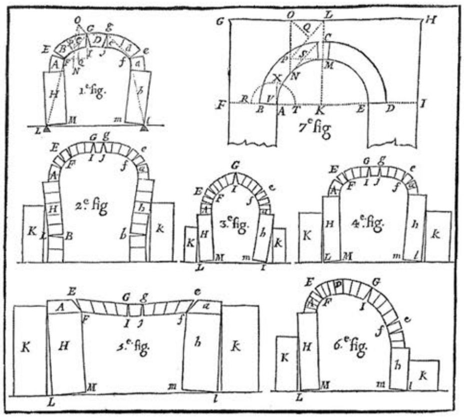
conclusion that an infinite load is needed at the springings. La Hire was aware of this and remarked that «[...] il n'est pas besoin de garder la proportion qu'on vient de déterminer pour la charge des voussoirs dans toute la rigueur, il suffit d'y avoir égard»[15]. Figure 4a shows the original drawing, and figure 4b represents the equilibrium of the second voussoir, of weight P_D, due to the thrust of the adjacent voussoirs which exert the thrusts F_L and F_O. The directions of the last two forces must be normal to the plane of joints and are fixed. The triangle CLO in figures 4a and 4b determine the weight P_D. In figure 4c, the triangle has been rotated 90° to make the sides parallel to the directions of the forces. By inspection, it will be evident for the reader that the triangle forms part of a polygon of forces with the pole at C, and radii CK, CL, CO, CP, etc.

In the case of the semicircular arch, La Hire imposed the condition that the trajectory of these thrusts must follow the locus of the centers of gravity, which forms a circumference near the middle line of the arch[16]. In the case of the plate-bande, this line is a horizontal one, and the trajectory of forces must deviate from the form of the intrados. However, the relative weights of the voussoirs remain defined by the rotated polygon of forces CKLOP (fig. 4d). It should be noted that the point E has been chosen arbitrarily and that there are infinite situations of equilibrium with the internal forces normal to the joints (as we shall see, Charles Augustin de Coulomb was the first to notice this).

Couplet was well aware that the trajectories of forces (what we now call lines of thrust) could deviate from the line of the intrados or the middle line. Without knowing it, he drew the first line of thrust in his «fig. II» (fig. 5a) – the concept of the line of thrust was formulated seventy years later by Thomas Young. In his «fig. VI» (fig. 5b), he tried to compute the total weight of the vault EATR, which is in equilibrium without friction. He realized, by simple geometrical reasoning, that the area of the arch is precisely the same as that of the plate-bande $\alpha\beta$TR, and that the weights of the voussoirs of the arch are equal

[15] La Hire, *Traité de méchanique*...cit., p. 318.

[16] The distance δ of the center of gravity from the middle-line is in most cases negligible, as δ is given by the expression, $\delta = (1/12)(t/r^2)$, where t is the thickness of the arch and r the radius of the intrados. See M. Milankovitch, «Theorie der Druckkurven», in *Zeitschrift für Màthematik und Physik*, 1907, n° 55, pp. 2-3.



6 Experiments by Danyzy on small gypsum arches to demonstrate the correct way of collapse of masonry arches; no sliding was observed. Note in «fig. 5» the pattern of cracks in the plate-bande (A. A. H. Danyzy, «Extrait du mémoire de M. Danyzy, sur la Poussée des Voûtes», in *Assemblée publique de la Société des sciences de Montpellier*, vol. VII, 1732, pp. 3-15).

to the corresponding ones in the plate-bande. As the direction of joints is the same, it follows that the auxiliary plate-bande is also in equilibrium. In words by Couplet: «Donc si l'intrados d'une Voûte est rectiligne et horizontale comme dans les Plates-bandes, son extrados doit être aussi rectiligne et horizontale; car alors les Voussoirs ou Claveaux seront entre eux dans la rapport des segments des ces lignes horizontales, comme ils doivent être pour faire équilibre entre eux»[17]. This is the only comment to this fundamental property.

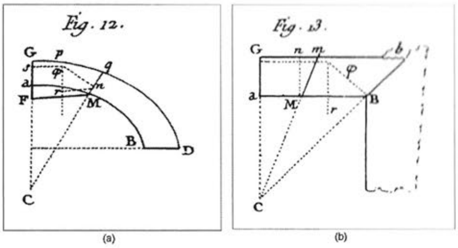
Danyzy, 1732

It was Augustin-Auguste-Hyacinthe Danyzy, mathematician, architect, astronomer, and he was one of the most active members of the Académie des Sciences de Montpellier, who first demonstrated the correct way of collapse of masonry vaults. The experiments by Danyzy were included by Amédée François Frézier in his treatise of stereotomy and, in this way, were widely diffused[18]. The tests of Danyzy opened the way to subsequent research activities following Couplet's approach, but with a firm experimental basis. Figure 6 reproduces the original plate inserted in the proceedings of the society of Montpellier in 1732. The model arches were made of gypsum on a small scale (there is no indication of their actual size). There were five different models: a semicircular arch «fig. 2», a pointed arch «fig. 3», a surbased arch «fig. 4», a plate-bande «fig. 5» and a rampant arch «fig. 6». The first figure «fig. 1» explains his theory of the thrust of arches and resumes the main traits of the experiments. It must have been an absolute surprise for the architects and engineers familiar with the no-friction theory to verify that in no case did sliding occur between the voussoirs. The tests gave an extraordinary support to Couplet's second memoir based on the impossibility of sliding.

The aim of Danyzy was, of course, to deduce a formula to calculate the depth of the buttresses. As he has worked as an architect, he was well aware of the necessity of simple rules. As he presented only the final result of his investigations, we cannot ascertain his

[17] Couplet, *op. cit.*, p. 94.

[18] A summary was published in 1732; see A. A. H. Danyzy, «Extrait du mémoire de M. Danyzy, sur la Poussée des Voûtes», in *Assemblée publique de la Société des sciences de Montpellier*, vol. VII, 1732, pp. 3-15. The complete *mémoire* was published in 1778; see Id., «Méthode générale pour déterminer la résistance qu'il faut opposer à la poussée des voûtes», in *Histoire de la Société Royale des Sciences établie à Montpellier*, vol. II, [1718-1745], pp. 177, 40-56. Frézier discussed the *mémoire* in extenso and inserted the plates (rather better drawn) in his treatise of stereotomy. See A. F. Frézier, *La théorie et la pratique de la coupe de pierres et des bois pour la construction des voûtes et autres parties des bâtiments civils et militaires, ou traité de stéréotomie à l'usage de l'architecture*, Strasbourg, Doulsseker, Paris, Guérin, vol. III, 1739, pp. 380-385, pl. CXI.



(a) (b)

7 Coulomb's study of the inclination of the joints for a vault of a given form to be in equilibrium without friction. (a) General case. (b) Application to a plate-bande (C. A. Coulomb, «Essai sur une application des régles de maximis et minimis à quelques problémes de statique relatifs à l'architecture», in *Mémoires de Mathématique et de Physique, présentés à l'Académie Royale des Sciences par Divers Savants et lus dans ses Assemblées*, vol. VII, 1773, pl. II).

degree of understanding. However, his masterful reply to Frézier when he enquired about the reason to ignore the height of the buttress, and other details of his method, have us believe that he was well ahead of his contemporaries. It is a pity that the book entitled *Application de la Statique à la construction des Bàtimens*, which he promised to publish at the end of his memoir, was never completed (or at least never published)[19].

Coulomb, 1773

The next contribution was done by Coulomb in a memoir presented to the Académie Royale des Sciences in 1773[20]. Like Couplet, Coulomb considers vault theory with and without friction, and remarks, also, that the results of the first hypothesis do not agree with practice and are therefore «d'une faible utilité». It is his study of vaults considering friction and cohesion which constitutes his fundamental contribution to vault theory, and has been the subject of numerous investigations[21].

Coulomb only studied the plate-bande in the hypothesis without friction and we will restrict our comments to this part. Two conditions must be fulfilled: the forces must be normal to the joints, and the force must be contained within the masonry (the points M,q and M,m in figures 7a and 7b). It should be noted that he explicitly considers the trajectory of internal forces normal to the joints (which he calls «ligne des resultants») to be able to move freely within the masonry and to be not constrained to follow the center line or the line of intrados[22]. In this, his approach differs with all previous authors, being more general, and permits him to pose a new problem (fig. 7a): given the lines of extrados and intrados, one needs to find the direction of the joints so that the voussoirs are in equilibrium[23].

For any given joint *M,q* forming an angle h with the vertical, the weight *P* of the part *GaMq* composed with horizontal thrust *A* applied at *s*, which is constant, must give a force normal to the joint. Therefore: $P=A\ (\cos h/\sin h)$. Coulomb, then, applied this condition to a plate-bande and obtained the same result as Couplet: all the joints must converge to the same center (fig. 7b). Subsequently, he added the second condition: the «ligne des

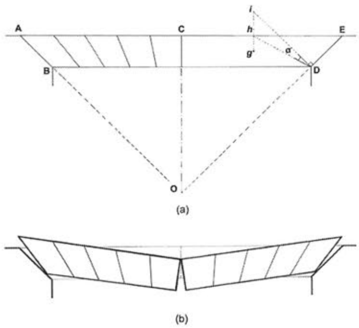
[19] Danyzy was the first to discover that a buttress of infinite height which support a finite thrust at the top, could be stable with a finite base. The letter was also included by Frézier in his treatise; however it appears that the brilliant reasoning of Danyzy was not duly appreciated (Frézier, *op. cit.*, pp. 370-372). For a detailed discussion of Danyzy's memoir see Huerta, *Arcos, bóvedas y cúpulas*...cit., pp. 332-338.

[20] C. A. Coulomb, «Essai sur une application des régles de maximis et minimis à quelques problémes de statique relatifs à l'architecture», in *Mémoires de Mathématique et de Physique, présentés à l'Académie Royale des Sciences par Divers Savants et lus dans ses Assemblées*, vol. VII, 1773, pp. 343-382.

[21] The best critical discussion, the facsimile, and an English translation, in J. Heyman, *Coulomb's Memoir on Statics: An Essay in the History of Civil Engineering*, Cambridge, Cambridge University Press, 1972.

[22] *Ibid.*, p. 81, remarks this point. Here again, we find an intuition of the concept of line of thrust and an implicit recognition of the possibility of drawing several (infinite) lines in equilibrium with the loads within the masonry.

[23] The problem was studied afterwards by other authors. See D. Aita, «Between geometry and mechanics: A re-examination of the principles of stereotomy from a statical point of view», in Huerta, ed., *Proceedings of the First International Congress on Construction History*...cit., pp. 161-170.



8 Collapse of a plate-bande of insufficient thickness in the absence of friction. A hinge forms in the middle and both ends slide upwards on the springer joints – with the friction coefficient $\geq \tan\alpha$ the plate-bande would not collapse (drawings by the author).

resultants» must be contained within the masonry. This imposes a limit to the thickness of a plate-bande, and Coulomb demonstrated that the normal to the surface of the springer, at its lowest limit, must cut the vertical passing through the center of gravity of the half plate-bande within the masonry. If this point of the intersection is above the line of extrados, the plate-bande will collapse, «la plate-bande se briseroit nécessairement»[24]. Coulomb gives no explanation of the mode of collapse, probably because he considers it evident. As a matter of interest, it has been sketched in figure 8. As the intersection of the vertical passing through the center of gravity g with the normal to the lower point of the springer (DE) is outside the masonry (point i in figure 8a), the thrust must take the direction hD which forms an angle α with the normal to the joint DE. As a consequence, a hinge forms at the keystone C and the joints slide upwards on the surface of the springers (fig. 8b).

In the part concerning the analysis of vaults with friction, Coulomb explained the correct analysis of any arch, including the plate-bande. But Coulomb only sketched the procedure with reference to a general joint in an abstract, mathematical way, and he did not give examples of application. Maybe for this reason, the fundamental memoir by Coulomb exerted almost no influence during the next forty years.

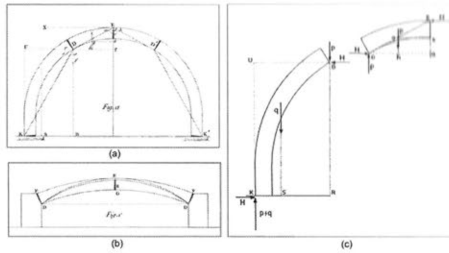
Boistard, 1800

In 1800, while working on the building of the bridge of Nemours, Louis Charles Boistard carried out systematic experiments on the collapse of vaults. He was trying to obtain a correct theory of vaults based on the direct observation of tests on model vaults of large sizes (circa 8 feet or 2,40 m)[25]. He criticized the no-friction theory and cited Couplet and Gaspard Clair François Marie Riche de Prony, but did not cite Danyzy or Coulomb. The experiments demonstrated that the collapse always occurs by forming hinges and that sliding does not take place. Boistard made 22 tests on arches of various forms[26]. In the final pages he outlined a theory of vaults. He considered that the collapse occurs by forming a four-bar mechanism and assumed that there are hinges at the crown and at the springings. Then, the problem is to find the exact location of the «joint de rupture». He established the

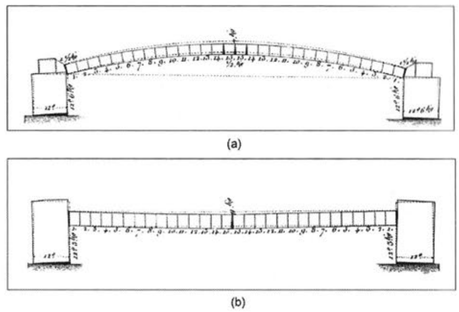
[24] Coulomb, *op. cit.*, p. 34.

[25] The manuscript was included in the collected essays by Lesage in 1810 (L. C. Boistard, «Expériences sur la stabilité des voûtes», in P. C. Lesage, ed., *Recueil de divers mémoires extraits de la bibliothèque impériale des ponts et chaussées a l'usage de MM. les ingénieurs*, Paris, F. Didot, 1810, vol. II, pp. 171-217, pl. XI-XVI, and was eventually published in book form in 1822 : L. C. Boistard *Recueil d'expériences et d'observations faites sur différents Travaux exécutés pour la construction du pont de Nemours, pour celle de l'arsenal et du port militaire d'Anvers, et pour la reconstruction du port de Flessingue ; dans lequel on a traité la théorie de l'équilibre des voûtes*, Paris, Feugueray, 1822).

[26] See Heyman, *Coulomb's Memoir on Statics*...cit., pp. 183-184.



9 Vault theory of Boistard. (a) Collapse of a typical vault. (b) Collapse of a surbased arch (L. C. Boistard, « Expériences sur la stabilité des voûtes », in P. Lesage, ed., *Recueil de divers mémoires extraits de la bibliothéque impériale des ponts et chaussées a l'usage de MM. les ingénieurs*, Paris, F. Didot, 1810, vol. II, pl. XVI). (c) Equilibrium at collapse after fig. a. (redrawn by the author).



10 Tests on model arches. (a) Surbased arch. (b) Plate-bande (L. C. Boistard, « Expériences sur la stabilité des voûtes », in P. Lesage, ed., *Recueil de divers mémoires extraits de la biblio-théque impériale des ponts et chaussées a l'usage de MM. les ingénieurs*, Paris, F. Didot, 1810, vol. II, pl. XVI).

equilibrium of the arch with hinges at K, D, E, D', K' (fig. 9a). The arch is divided into four parts, with weights q (KD, K'D') and p (ED, ED'), and it follows, taking moments with respect to K in figure 9c, that when the arch is in strict equilibrium:

$$p\times\frac{kQ}{EQ}\times\frac{DQ}{EQ}\times KU=p\times KR+q\times KS.$$

The left member of the equation represents the moment of the horizontal thrust H, i.e., $H=p\times\frac{kQ}{EQ}\times\frac{DQ}{EQ}$ as can be easily deduced by the equilibrium of the upper part ED in figure 9c.

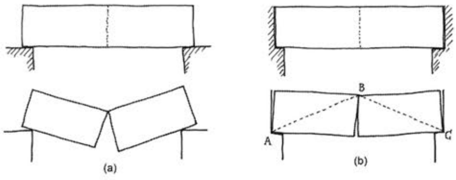
Boistard was aware that the real problem consisted in locating the joint of rupture, but he did not try to solve the problem, as he realized that « le calcul sera souvant fort long, à cause des quantités trascendentales qui naissent du cercle »[27].

For surbased segmental arches (fig. 9b) or plate-bandes, Boistard pointed out that the joint of rupture is always at the springing of the arch, as demonstrated by his tests (fig. 10). Besides, the center of gravity of the semi-arch will be very nearly in the middle of the semi-span DQ. Then, $Dh\div kQ = DQ\div EQ$ and the horizontal thrust can be expressed as: $H=p\times\frac{Dh}{EQ}$.

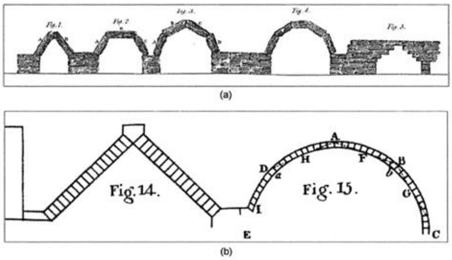
If we call the span s (= $4\times Dh$), $EQ = c$, and we may express the thrust as $H=\frac{W}{8}\times\frac{s}{c}$, where W (= $2\,p$) is the total weight of the surbased arch or plate-bande. This simple expression renders it possible to calculate correctly and rapidly the horizontal thrust of surbased arches or plate-bandes[28]. It was widely used during the second half of the 19[th] century by

[27] Boistard, « Expériences sur la stabilité des voûtes... » cit., p. 201. Gauthey follows the approach of Boistard in his treatise on bridges, arriving at the same formula. He agrees with him that « le calcul est presque impracticable pour les arches en pleine cintre » (E. M. Gauthey, *Traité de la construction des ponts (publié par M. Navier)*, Liège, Leduc, vol. I, (1809), 1843, p. 242. An English translation of part of the treatise is included in J. Weale, *The Theory, Practice and Architecture of Bridges of Stone, Iron, Timber and Wire*, London, Architectural Library, 1839-1843, vol. I.

[28] Boistard says explicitly that the horizontal thrust at the springings: « [...] est égale au huitième du poids de la voûte entière, multiplié par le rapport de l'ouverture à la flèche augmentée de l'épaisseur de la voûte. Dans les plate-bandes, la pression à la clef est égale au huitième du poids entier multiplié par le rapport de l'ouverture à l'épaisseur [...] ». (Boistard, « Expériences sur la stabilité des voûtes... » cit., p. 199). A few lines before, he makes a numerical mistake (he misses a factor of ½), and calculates the thrust at the mid-keystone (point E in fig. 8) as twice this quantity. He was, then, apparently unaware of the constant value of the horizontal component of the thrust.



11 Graphical interpretation of Robison's remarks on the behavior of a broken lintel. (a) Simply supported broken lintel. Collapse is immediate. (b) Broken lintel simply supported with lateral masonry which precludes the movement of points A and C. For the lintel to collapse, a large displacement of the supports is necessary, 2AB = span AC (drawings by the author).



12 (a) Simple polygonal and false arches. (b) Left («fig. 14»), arch formed by two plate-bandes and rectilinear transmission of thrusts within a normal arch ; right («fig. 15») rectilinear transmission of a thrust within a circular arch to explain the mechanism of collapse (J. Robison, «Article Arch», in *Supplement to the third edition of the Encyclopaedia Britannica*, Edinburgh, Thomson Bonar, 1801, pl. I-II).

practicing engineers and architects[29]. However, as we shall see, the origin is usually attributed to Moseley in 1843.

Young, 1807

The next advance in the study of plate-bandes is contained in an article by Young published in 1807 under a pseudonym[30]. The article has the title «Remarks on the Structure of covered Ways, independent of the Principle of the Arch in Equilibrium, and on the Best Forms for Arches in Buildings»[31]. In it, Young considered first the problem of «false arches», formed by successive stones in cantilever (he cites the gate of the treasury of Atreus at Mycenae), and «angle arches», formed by two inclined stones or plate-bandes; he then considered usual horizontal plate-bandes (see fig. 13, below). Eventually, the research led him to fundamental conclusions on arch design. As his predecessors, he considered the matter with and without friction. Of course, the problem compelled him to abandon the English theory of equilibration, first proposed by Hooke in 1675 – «As hangs the flexible line, so, but inverted, will stand the rigid arch»[32]. It seemed impossible to adapt this approach to flat arches, and, as a consequence, the plate-bande was not studied by English engineers before Young.

There is however an important exception: John Robison. In his article «Arch» for the Supplement to the third edition to the *Encyclopaedia Britannica*[33], Robison is very

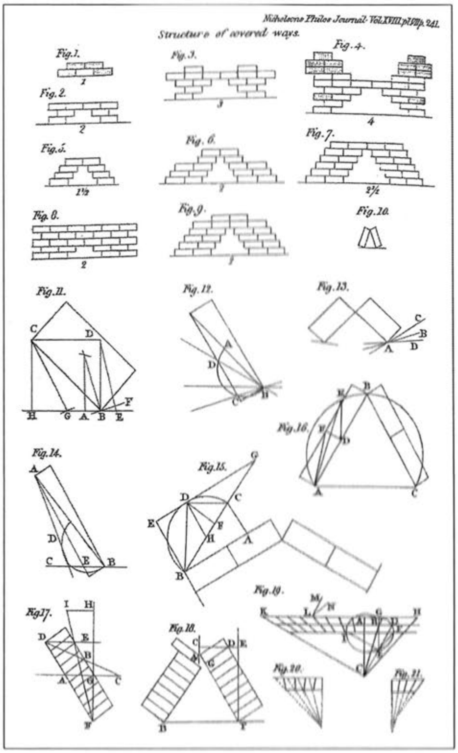
[29] Substituting $W = ql$, we arrive at the modern formulation : $H = qs^2/_8c$.

[30] Young published most of his contributions anonymously or under a pseudonym, as he was afraid that his scientific activity would damage his reputation as a physician. On his life and works is still fundamental : G. Peacock, *Life of Thomas Young*, London, J. Murray, 1855. A recent biography in A. Robinson, *The Last Man Who Knew Everything: Thomas Young, the Anonymous Polymath Who Proved Newton Wrong, Explained How We See, Cured the Sick and Deciphered the Rosetta Stone*, Oxford, Oneworld Publ., 2006.

[31] T. Young, [signed APSOPHUS], «Remarks on the Structure of covered Ways, independent of the Principle of the Arch in Equilibrium, and on the Best Forms for Arches in Buildings», in *A Journal of Natural Philosophy, Chemistry and the Arts*, vol. XVIII, 1807, pp. 241-250, pl. VII. Republished in T. Young, *Miscellaneous Works*, G. Peacock ed., London, J. Murray, 1855, vol. II, pp. 179-189.

[32] J. Heyman, *Structural analysis : a historical approach*, Cambridge, Cambridge University Press, 2008, p. 79.

[33] J. Robison, «Arch», in Supplement to the third edition of the Encyclopaedia Britannica, Edinburgh, Thomson Bonar, 1801, pp. 21-38, 4 plates. (Reprinted in America as : Supplement to the *Encyclopaedia of Arts, Sciences, and Miscellaneous Literature*, Philadelphia, Dobson, 1803. We are using this reprint). It was incorporated in the 4th to 8th editions of the *Encyclopaedia*, and republished with minor alterations as : J. Robison, «On the construction of arches», in *A System of Mechanical Philosophy*, Edinburgh, J. Murray, 1822, vol. I, pp. 616-660, pl. X.



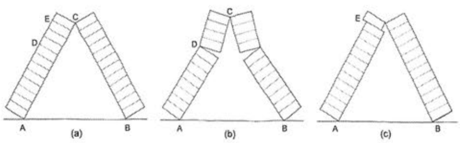
13 T. Young, *Structure of covered ways* (T. Young, «Remarks on the Structure of covered Ways, independent of the Principle of the Arch in Equilibrium, and on the Best Forms for Arches in Buildings», in *A Journal of Natural Philosophy, Chemistry and the Arts*, vol. XVIII, 1807, pl. VII).

critical to the «popular» equilibration theory and was the first English author to consider the fundamental role of friction in the stability of arches. With respect to lintels and plate-bandes, we find in the article a most interesting comment on the behavior of a broken lintel: «The two parts of the broken lintel seem to push the piers aside in the same manner that two rafters push outwards the walls of a house [...] If the piers cannot be pushed aside (as when the arch abuts on two solid rocks), nothing can press down the crown which does not crush the stone»[34]. This means that a broken lintel simply supported will fall down (fig. 11a), but if its extreme joints can resist a thrust, it will behave as a flat arch or plate-bande (fig. 11b).

Also, an arch formed by two inclined stones will be stable (see «fig. 1», in figure 12a) or an arch subject to a vertical point load with joints normal to the thrusts («fig. 14», in figure 12b). Robison goes further, and argues that due to the effect of friction, a straight thrust can be transmitted within the masonry, deviating from the normal to the joints. He used this observation to interpret the collapse of a real bridge and the tests realized by him on small arch models («fig. 15», in figure 12b). Robison did not cite any of the French authors on arch or plate-bande theory.

The article by Robison influenced Thomas Young on ideas on arch behavior. His criticism to the equilibration theory was the first step to «free» the line of thrust from the strait-jacket of the intrados. This is evident in the comments on arches made by Young in the Lecture XIV «On Architecture and Carpentry» of his *Course of Lectures on Natural*

[34] Robison, «Arch» cit., p. 29.



14 (a) Modes of collapse of two inclined brick plate-bandes. (b) A joint opens at D and the arch collapses with an upwards movement of the « keystone » C. (c) One of the upper bricks E slides upwards or, conversely, the block EA slides downwards (drawings by the author).

Philosophy and the Mechanical Arts[35]. The Lectures were published the same year as the above-cited article, but they must have been written before, taking into account the long process of book edition. Furthermore, the article represents a complete break and a considerable advance in arch theory.

The article has only one plate, reproduced in figure 13. It has a naïve aspect, and the simple drawings look like a children's wooden block toy. It may be for this reason, that the article has been ignored until now in every history of arch theory. Another reason may be the anonymity of the author and, finally, the obscure prose and the absence of clear explanations. Indeed, it is quite difficult to imagine that in barely ten pages, Thomas Young was breaking new ground and making a giant step in arch theory, which eventually led to his formulation of the line of thrust theory and its successful application to a complex case of arch analysis[36].

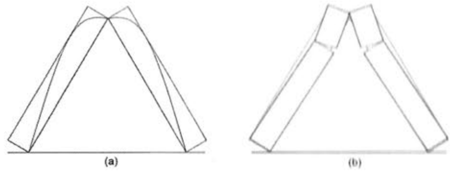
In what follows, we give a brief summary of its content[37]. The article begins by considering different dispositions of blocks or bricks forming false arches without friction (« fig. 1-9 », in figure 13). Young then considered the problem of two bricks leaning one against the other; in the absence of friction and on a horizontal basis, the only disposition is shown in « fig. 10 »; the vertical passing through the center of gravity of the bricks must pass through the points of support. In « fig. 11 », he studied the inclination of the planes of support in order for two blocks to be stable ; the plane AF must be normal to the direction of the reaction DE. Subsequently, he considered that the support is a plane coinciding with the inferior joint of the two blocks. In this case, there exist different possible inclinations of the blocks, supported without friction on the plane inferior joints, and Young gives a correct geometrical construction which shows the two stable leanings, CB and DB in « fig. 12 », and discusses the form of collapse for other directions. In « fig. 13 », Young explains the limits of the inclination of the support planes for a certain angle of friction; the planes may deviate from the plane of support without friction, « fig. 11 », precisely the angle of friction. Then, he considered the problem of two blocks with inferior joints on plane supports, taking friction into account, « fig. 14 », and went on to the much more complicated problem of four blocks, « fig. 15 » and « fig. 16 ». Eventually, he arrived at the general problem of two inclined plate-bandes formed by any number of bricks and leaning against each other with any inclination.

35 T. Young, *A Course of Lectures on Natural Philosophy and the Mechanical Arts*, London, J. Johnson, 1807, vol. I, p. 157. The Lectures were delivered in 1803-1804 at the Royal Institution of London. The first volume contains the texts of the Lectures without mathematical or geometrical explanations. The second volume contains the proofs of the principal statements, a bibliography of more than 20 000 titles, and a collecion of essays published until then by Young. See N. G. Cantor, « Thomas Young's lectures at the Royal Institution », in *Notes and Records of the Royal Society of London*, vol. XXV, 1970, pp. 87-112.

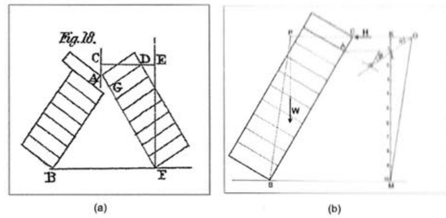
36 See S. Huerta, « Thomas Young's theory of the arch : His analysis of Telford's design for an iron arch of 600 feet span », in S. Huerta, ed., *Essays in the history of the theory of structures, in honour of Jacques Heyman*, Madrid, Instituto Juan de Herrera, Centro de Estudios Historicos de Obras Públicas y Urbanismo, 2005, pp. 189-233, and S. Huerta, « The First Thermal Analysis of an Arch Bridge : Thomas Young 1817 », in *First International Conference on Advances in Bridges Engineering. Bridges – Past, Present and Future*, London, Brunel University Press, 2006, pp. 18-29.

37 A detailed analysis of the article would be too long for the present publication and will be the subject of a subsequent paper.

15 Collapse of an arch formed by plate-bandes inclined 60° by pure rotation of the parts, considering sliding impossible. Note the discontinuity of the curvature of the line of thrust at the mi (drawings by the author).



16 Collapse by sliding of the upper brick of two inclined plate-bandes. (a) Young's drawing to calculate the equilibrium by the principle of virtual work (Th. Young, « Remarks on the Structure of covered Ways, independent of the Principle of the Arch in Equilibrium, and on the Best Forms for Arches in Buildings », in *A Journal of Natural Philosophy, Chemistry and the Arts*, vol. XVIII, 1807, pl. VII). (b) Graphic statics analysis (drawing by the author).



Young's reasoning constitutes an amazing *tour de force*, as he had to invent a new type of analysis to solve an apparently simple problem. Here, Young's famous obscurity and ingenuity is at its best[38]. The two plate-bandes of bricks can collapse in two ways : by rotation of the lower part on the upper edge of the joint of rupture and the higher part revolving in the contrary direction ; or by rotation of the lower part and the higher part sliding upwards (fig. 14).

For the first way of collapse (fig. 14b) Young provided a geometrical construction, « fig. 17 », which appears to be incorrect. Then, he stated that for an equilateral intrados, « 15 common bricks[39] on each side will stand, but 16 will give way at the sixth joint from the summit »[40]. That gives a proportion for the plate-bandes in the collapse of 8,75/48, nearly 2/11, and the joint of rupture at 6/16 from the top. The exact calculation, resolving a transcendental equation to obtain the limit thickness, gives very nearly a proportion of 1/6 and the position of the joint of rupture at circa 1/4 of the length from the top (fig. 15). The difference between Young's solution and the correct solution is small and can be reduced considering discrete elements (the continuous line of thrust in the figure has been calculated for elements of differential thickness).

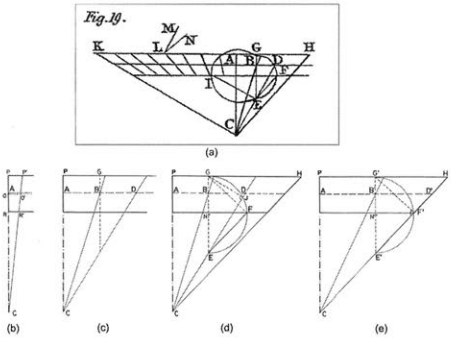
For the other way of collapse, Young presented what is maybe the first published drawing of mixed collapse by rotation and sliding[41], « fig. 18 » (fig. 13, reproduced in fig. 16a). Young arrived at the correct conclusion that the collapse will occur by the sliding of one of the first bricks, and, again for an equilateral intrados, he stated that this will occur for

[38] Peacock describes Young's method of attacking a problem : « [...] the extraordinary capacity which he possessed of solving the most difficult problems in the applications of mathematics to natural philosophy, by processes apparently the most inadequate to the purpose. He never confined himself to the beaten track of a systematic investigation. We find in his writings no symmetrical formulae or analytical refinements. There is no seeking after generalities, when the particular question which he has in hand does not require them; whilst every expedient is freely resorted, however irregular and unusual, if it serves the purpose which he has in view. Important and difficult steps are passed over as manifest, terms are neglected as insignificant, analogies take the place of proofs, and we are surprised to find ourselves at the end of an investigation, even within the limits of space which would commonly be deemed hardly sufficient to master the difficulties which we met at the beginning » (Peacock, *Life of Thomas Young*...cit., p. 416).

[39] It can be deduced from the text that the brick has a thickness of 3 in, and is 8,75 in long. The breadth does not enter into the calculations.

[40] Young, « Remarks on the Structure of covered Ways... » cit., p. 247.

[41] The study of mixed collapse mechanisms was made also by the Spanish engineer Joaquín Monasterio circa 1805 in his, unpublished, « Nueva teórica sobre el empuje de las bóvedas » [New theory on vault thrusts]. See S. Huerta, F. Foce, « Vault theory in Spain between the XVIII[th] and the XIX[th] century : Monasterio's unpublished manuscript "Nueva teorica sobre el empuje de las bóvedas" », in Huerta, ed., *Proceedings of the First International Congress on Construction History*... cit., pp. 1155-1166. The manuscript will be published as : J. Monasterio, *Nueva teórica sobre el empuje de las bóvedas*, ed. S. Huerta, F. Foce, Madrid, Instituto Juan de Herrera.



17 (a) T. Young, *Structure of covered ways*, detail (T. Young, « Remarks on the Structure of covered Ways, independent of the Principle of the Arch in Equilibrium, and on the Best Forms for Arches in Buildings », in *A Journal of Natural Philosophy, Chemistry and the Arts*, vol. XVIII, 1807, pl. VII). (b) Geometrical data : thickness and center of joints. (c–e) Steps of the geometrical construction (drawings by the author).

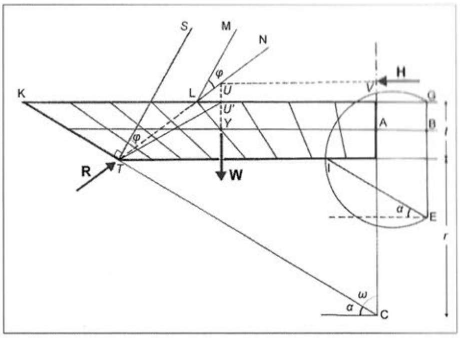
plate-bandes of 9 bricks. To arrive at this conclusion, he used the principle of virtual work. The result is, again, nearly correct : 9 bricks will be stable but 10 bricks will collapse. The solution may be understood easily by inspecting figure 16b, in which a graphic statics calculation has been made. The angle of friction ϕ[42] determines the maximum inclination of the force 01. Since the line OL passes under the point 1, the upper brick will not slide. It is evident that the upper brick is very near the limit, and possibly the rounding of Young's calculation is the origin of his error.

Young's paper has a Postscript in which he extracts consequences of his new discoveries. First he studies the horizontal plate-bandes : «The equilibrium of the flattened arches, commonly placed over windows, may be determined in a similar manner, the principles being the same [...]. Supposing the blocks without friction and of equal height, if their divisions converge to one point, the lateral thrust will be equal throughout, and the whole will remain in equilibrium, provided that the ends do not slide outwards». The last remark alludes to Coulomb's second condition, that the vertical passing through the center of gravity must cut the normal at the lower end of the springer joint within the masonry. But he gives no explanation, and proceeds to explain a geometrical construction to find the limit span for a plate-bande when the thickness and the center of the joints are given. The algebraic solution to the problem leads to a third-degree equation[43].

Young's geometrical method is completely correct and much more simple: «In order to find the breadth which is within this limit, let the horizontal line A B (« fig. 19 », fig. 17a) pass through the center of gravity of the blocks, draw any line CB from the centre of divergence C, make BD = A B, join CD, and let the vertical line BE meet it in E ; then EF, drawn to the intersection of the semicircle EFG with the lower termination of the blocks, will show the direction of· the abutment *d*, which will afford an equilibrium; and CH parallel to it will determine the greatest breadth that will stand»[44].

The construction shows great ingenuity. First, the point A, marking the position of the all the centers of gravity of the wedges with joints converging in C, is obtained (fig. 17b) – considering the the point A at the center involves a negligible error[45]. Then, an arbitrary point B is chosen on the horizontal line passing through A, and another point

[42] Young is taking as friction coefficient 9/20, so that the angle of friction $\phi = 24{,}23°$.
[43] See G. Venturoli, *Elementi di Meccanica e d'Idraulica*, Milano, Giusti, vol. I, (1807), 1817, p. 284.
[44] Young, « Remarks on the Structure of covered Ways... » cit., p.248.
[45] The distance is given by the same expression for normal curved arches $\delta = (1/12)(t/r^2)$. With reference to figure 17a, δ = AO, t = PR and r = RC. The demonstration may be easily obtained from that given by Milankovitch, « Theorie der Druckkurven... » cit., pp. 2-3.

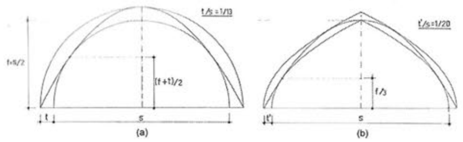


18 Equilibrium of the half plate-bande of the left side of fig. 17a, designed following Young's rule. Note that the equilibrium is not possible for masonry, as the horizontal thrust H is without the masonry. Probably, Young considered that the upper wall could transmit the internal forces functioning, in fact, a thickening of the plate-bande. For the particular case of $\phi = \alpha = 30°$ the construction is correct (drawing by the author).

D at the same distance from B (fig. 17c). Then, point B is the center of gravity of the wedge defined by the joints A and D. The vertical line passing through B cuts the extrados at G and the line CD at E. The semicircle with diameter GE (center at N) permits drawing the line GJ normal to the joint D. This line marks the direction and point of application of the thrust applied at J which will maintain the wedge AD in equilibrium with a horizontal thrust at P. Now, we want to obtain the maximum span which will allow the equilibrium without friction. The point F of the intersection of the semicircle with the intrados gives us the clue : the line EF marks the maximum inclination of the abutment joint, as is evident by the construction in figure 17e (the diameter GE = GE'). However, Young remarks : «But since the blocks thus disposed, and supporting a wall, cannot slide away without displacing the superincumbent weight, *the whole wall may be considered as adding to the height of the blocks*, and the stability in every case that can occur in practice, must be complete»[46]. That means, in fact, that the thrust can go out of the plate-bande violating Coulomb's second condition. Here, Young expressed his concern for practical considerations which can simplify the analysis.

In the case of friction, Young proposed an approximate rule: to consider the inclination of the springer joint simply as the angle of friction. However, the rule he gave is (fig. 17a) : «If we wish to estimate also the effects of friction, let the segment EIG contain a right angle diminished by the angle of repose, then CK, parallel to EI, will be the direction of the abutment which will secure the blocks from sliding outwards, with the assistance of the force of friction»[47]. The rule is not general and can be unsafe for certain angles of friction ϕ and the proportion between the distance of the center of convergence C to the intrados, r, and the thickness t, ($n = r/t$). Indeed, for the proportions of the drawing in figure 17a ($n = r/t = 2{,}7$), the maximum angle of aperture ω is less than $(\pi/2 - \phi)$, Young's rule. It is evident in figure 18 that the vertical passing through the center of gravity Y should cut the line TN, which represents the maximum inclination of the reaction R from the normal to the joint KT, in a point U within the masonry, but U is above the extrados. Young mentions in the paper two different values for the coefficient of friction, 0,5 and 9/20, corresponding to angles of friction ϕ of 26,6° and 24,2°, and in figure 18 the greater value has been used. Both are conservative values. For $\phi = 30°$, the rule is very nearly true for the proportions of the figure ($\omega = 59°$), but unsafe for other proportions. As it often happens with Young, we cannot know if he is giving a particular result or if it is an error. We will be inclined to

[46] Young, «Remarks on the Structure of covered Ways...» cit., p. 248. The italics is from the author.
[47] *Ibid.*



19 Limit arches for a parabolic line of thrust. (a) Semicircular. (b) Pointed, «Tudor», arch formed by two parabolic segments (drawings by the author).

believe the first option as the problem is mathematically trivial in comparison with others treated in the same paper[48]. In any case, he pointed above that the upper wall could transmit the internal forces functioning, in fact, as a thickening of the plate-bande.

The fundamental consequence of all the previous work was to realize that the effect of friction permits, indeed, to free the «curve of equilibrium», or line of thrust, from the form of the intrados of the arch. Moreover, Young extracted the consequences for arch design. First, he remarked that the curve of equilibrium must be contained within the arch: «The size of the blocks must be such, that the curve of equilibrium, under the pressure actually produced by the walls may be everywhere included within their substance, and even without coming very near their termination»[49]. He, then, remarked that the form of the curve for an uniform load is a parabola and that this will also be its form if the wall supporting the arch has a considerable height: «Supposing the height of the wall supported by the arch to be very considerable in proportion to that of the arch itself, the curve of equilibrium must be very nearly a parabola» and concluded that, for the case of a high wall: «In order therefore to find whether the size of the blocks is sufficient, describe a parabola through the summit and the abutments; and if it pass wholly within the blocks, they will stand; provided however that their joints are either perpendicular to the curve, or are within the limits of the angle of repose on either side of the perpendicular»[50]. Young, then, discussed the form of the line of thrust as a function of the load and stated, correctly, that for flat arches or plate-bandes, an arc of circle is more approximate. He also remarked that the thrust will diminish with the height of the arch.

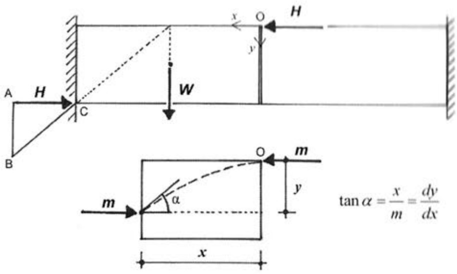
He, then, applied this statement to calculate the limit thickness of a semicircular arch and of a pointed arch formed by two parabolic segments, both of the same height: «[...] supposing the wall very high, the depth of the arch stones of a semicircular arch must be at least 1/13 of the span, in order that the arch may stand; but that of the stones of a Gothic arch, composed of two parabolic segments, may be less by one twentieth; the parabola of equilibrium touching in this case the internal limit of the arch at 23/100 [sic] of its whole height above the abutments»[51].

In barely a page of his Postscript, Young advanced the fundamental ideas of masonry arch analysis: the concept of line of thrust, its relation with the load and the concept of geometrical safety.

Young, 1817

After the publication of his *Lectures* in 1807, Young seemed to have lost interest in the Mechanical Arts. He signed only his contributions on medicine and those related to the

[48] The formula relating ω, ϕ and n, can be found in A. Ritter, Lehrbuch der technischen Mechanik, Hannover, Rümpler, 1865, p. 292.
[49] Young, «Remarks on the Structure of covered Ways...» cit., p. 249.
[50] *Ibid.*
[51] *Ibid.* The value of 23/100 is incorrect. As it turns out, the exact value is 33/100, and we may assume an error of transcription from the manuscript. The construction of a Tudor arch by two parabolic segments was first proposed in W. Halfpenny, *The Art of Sound Building*, S. Aris, London, 1725, pl. II, fig. 13, and popularised by P. Nicholson, *The Principles of Architecture*, London, Barfield, (1795), 1809, vol. II, p. 6, pl. LXII. Of course, it is an invention, and Robert Willis criticized sharply these «fanciful hypotheses, which «[...] produce curves for the ribs totally different from the genuine one, can answer no purpose but that of destroying the mediaeval character of the work» (R. Willis, «On the Construction of the Vaults of the Middle Ages», in *Transactions of the Royal Institute of British Architects*, vol. I, part II, 1842, p. 22. No doubt Young used it to easy the mathematical calculations).



20 Equilibrium of two blocks leaning against each other and a differential equation of the line of thrust (drawings by the author).

Royal Society. However, his reputation in the field of applied mechanics was great and, for example, in 1814, he was required to write a technical report for the Board of Admiralty on ship design[52]. In fact, the same year McVey Napier, at the time editor of the *Encyclopaedia Britannica*, wrote to him asking him to contribute in a variety of topics. Young refused at first, but eventually, a year a half later, yielded under the condition of anonymity. All in all, he wrote 61 articles between 1816 and 1824 (of which 45 were biographical) for the *Encyclopaedia*[53]. Some of these articles were completely original and contained substantial advances on the topic. This is the case of the article «Bridge», finished in May 1817, and published the same year in the second volume of the *Supplement* to the Encyclopaedia Britannica[54].

The contribution to arch theory contained in this fundamental article has been discussed in full elsewhere[55]. We will discuss only Young's contribution to plate-bande analysis. He systematized and expanded the findings and intuitions of the previous work. He defined without any ambiguity the concept of line of thrust, «curve of equilibrium», and obtained the mathematical expression for different cases. In particular, he stressed the importance of friction which allows the line of thrust to move freely within the masonry, provided that the thrust inclination was never greater than the angle of friction.

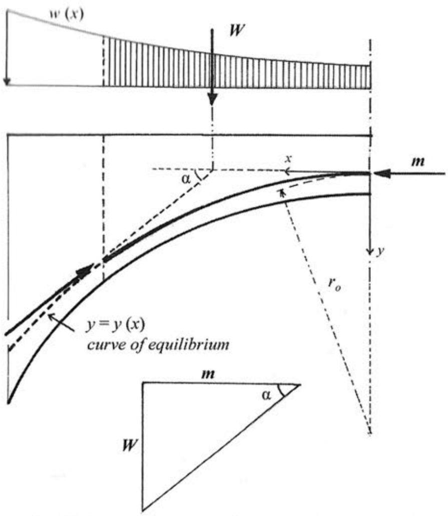
His analysis begins, precisely, with the plate-bande. He first considered the equilibrium of two parallelepipeds abutting one against the other (the same idea as Robison, (fig. 11)) : «If two equal parallelepipeds be supported each at one end, and lean against each other at the other; so as to remain horizontal, the curve of equilibrium, representing the general effect of the pressure transmitted, through them, will be of a parabolic form». Young remarked that the thrust at the center, where the two blocks meet, must be horizontal, and at the ends inclined, being the resultant of the horizontal thrust and the weight of the block. To study the transmission of internal forces he imagined the two blocks divided by vertical planes and observes that «it is evident that the force exerted at any of these sections, by the external portions, must be sufficient to support the lateral thrust and the weight of the

[52] T. Young, «Remarks on the employment of oblique girders and on other alterations in the construction of ships», in *Philosophical Transactions of the Royal Society*, 1814, vol. CIV, pp. 303-336 (reprinted in Young, *Miscellaneous Works*...cit., pp. 535-563).

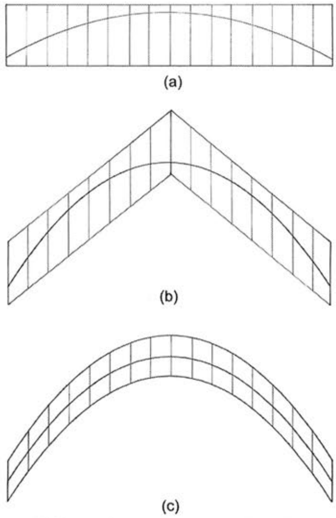
[53] See A. Wood, F. Oldham, *Thomas Young. Natural philosopher, 1773-1829*, Cambridge, Cambridge University Press, 1954, pp. 256-271. See also, R. Yeo, *Encyclopaedic Visions. Scientific Dictionaries and Enlightenment Culture*, Cambridge, Cambridge University Press, 2001, pp. 264-270.

[54] T. Young, «Bridge», in *Supplement to the fourth, fifth and sixth editions of the Encyclopaedia Britannica*, Edinburgh, A. C. Thomas, (1817), 1824, vol. II, pp. 497-520, pl. XLII-XLIV. The article was signed O.R. Only after the publication of all the volumes was completed in 1824, just five years before his death, did Young agree to making his contributions public. The authorship of the article aroused great curiosity among some eminent engineers like Rennie. See Wood, Oldham, *op. cit.*, p. 260. Part of the article was included in G. Peocock, ed., *Miscellaneous Works of the late Thomas Young*, London, J. Murray, 1855, vol. II, pp. 194-247, pl. III.

[55] S. Huerta, «Thomas Young's theory of the arch...» cit.



21 Equilibrium of a segment of a symmetrical vault with vertical plane joints (drawings by the author).



22 Different forms of arches with uniform vertical thickness with one parabolic line of thrust drawn. (a) Plate-bande or « flat arch ». (b) Triangular arch. (c) Parabolic arch (drawings by the author).

internal portions; and its inclination must be such that the horizontal base of the triangle of forces must be to the vertical perpendicular as the lateral thrust to the weight of the internal portion; or, in other words, the lateral thrust remaining constant, the weight supported will be as the tangent of the inclination ». Taking as origin the point where the block meets, x as abscissa and y as ordinates, then, the inclination dy/dx at a distance x, must be proportional to the weight of this part, i.e., to x, then, $\tan\alpha = \frac{x}{m} = \frac{dy}{dx}$, and integrating, $my = \frac{1}{2}x^2$ where m is proportional to the horizontal thrust H (fig. 20). The line of thrust is, consequently, a parabola. As we have seen, this result was already stated, without demonstration, in his paper of 1807.

Young returned again to the plate-bande after having deduced the general equation of the line of thrust for a general load, considering a vertical plane of joints. He considered the equilibrium of a symmetrical vault subject to a height of matter $w(x)$. The equilibrium of a part of the vault at a distance x of the keystone gives now: $W = \int w\,dx = m\frac{dy}{dx}$ where the integral represents the volume of material, and m is a quantity proportional to the horizontal thrust at the keystone.

If the load is uniform ($w = t$), then, the left integral gives tx, and the equation of the line of thrust is a parabola, $my = \frac{t}{2}x^2$

Therefore, the line of thrust of any arch which presents a uniform vertical thickness is a parabola. Quoting Young: «The uniformity of the load implies that the superior and inferior terminations of the arch, commonly called the extrados and intrados, should be parallel: but it is not necessary that either of them should be parabolic, unless we wish to keep the curve exactly in the middle of the whole structure. When the height of the load is very great in proportion to that of the arch, the curve must always be nearly parabolic, because the form of the extrados has but little comparative effect on the load at each point. A parabola will therefore express the general form of the curve of equilibrium in the flat bands of brick or stone, commonly placed over windows and doors, which, notwithstanding their external form, may very properly be denominated flat arches»[56]. In figure 22, some of the arches analyzed by Young have been drawn, showing a parabolic line of thrust : the plate-bande, the «triangular» arch formed by two plate-bandes and a parabolic arch.

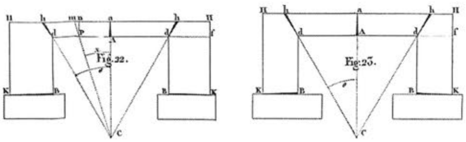
Young considered, in the last case of the parabolic profile, that the line of thrust had to pass through the middle of the joints, which is obviously impossible in the other two cases. In fact, in his analysis of arch bridges contained in the article «Bridge», he begins by having the line of thrust pass through the middle of the extreme joints and the keystone, but permits the line of thrust to move freely in the case of a point load acting on the bridge, adjusting the deformed thrust line within the thickness of the arch. Young, then, did not assume that any of the lines of thrust was the «actual» line of thrust, but contented himself finding one that satisfies the requisite of the material, which must work in compression both in masonry and cast iron, and verifying that the stresses do not approach the crushing strength of the material, i.e., the line of thrust does not approach too much the intrados or extrados in any point.

The form of the line of thrust depends on the family of planes selected (in the triangular arch the line changes if we consider joints normal or vertical to the intrados, fig. 15a and fig. 22b) ; the expressions deduced are correct under the assumption of vertical planes, which makes the algebraic work much more easier. Young was aware of this and remarked, that, in the case of a usual plate-bande with convergent (instead of vertical) planes of joint, the line of thrust varies and approximates more to an arc of circle than to a parabola. In any case, from a practical point of view, the differences are negligible, in particular for surbased arches.

We have shown that Young had a deep understanding of arch behavior, being several decades in advance as opposed to his contemporaries. He developed a complete theory of the arch and successfully applied it to the analysis of Telford's design for an arch of 600 feet over the Thames, and also to Blackfriar's, Waterloo and Southwark bridges. He not only computed the thrust, but considered the effect of a point load and the increment of thrust due to changes of temperature. With respect to plate-bandes, he was the first to correctly treat the analysis of internal forces and give the expression of the line of thrust.

His work in structural engineering was not duly acknowledged by his contemporaries, at least from a scientific point of view. In fact, no English engineer of the first quarter of the 19th century had the scientific background to free their thinking from the strait-jacket of conventional «equilibration theory». It is true that Young published anonymously (until 1824), but his article «Bridge», published first in 1817, remained in the *Encyclopaedia Britannica* until the 8th edition of 1854 (the 9th edition was published in 1875). The main reason may be that it was extremely difficult to understand his reasoning in his time (and it remains quite difficult !) because he had invented a frame of mind of his own, to attack completely new problems. We have documentary evidence; when writing the article he asked John Rennie for information on Waterloo and Southwark bridges. From the comments of John Rennie in his autobiography[57] and of his son George Rennie in several papers[58], it is evident that they were unable to understand the immense theoretical advance made

[56] Young, «Bridge» cit., p. 502
[57] J. Rennie, *Autobiography of Sir John Rennie*, London, Spon, 1875, p. 9.
[58] G. Rennie, «Account of experiments made on the strength of materials… in a Letter to Thomas Young… Read February 12th, 1818», in *Philosophical Transactions of the Royal Society of London*, 1838, vol. I, part I, pp. 118-136, pl. VI-VII, and G. Rennie, «On the expansion of arches», in *Transactions of the Institution of Civil Engineers*, vol. III, part III, 1842, pp. 201-218.



23 Collapse analysis of two plate-bandes of different thicknesses. The thicker plate-bande requires fewer buttresses (J. V. Audoy, «Mémoire sur la poussée des voûtes en berceau», in *Mémorial de l'Officier du Génie*, 1820, nº 4, pp. 1-96, pl. V-VI).

by Young[59]. Eventually, after the 1830's, the influence of the French authors, particularly Claude Louis Marie Henri Navier, who presented for the first time a complete theory of structures in the modern sense, brushed away most of Young's influence in structural matters. The Kelland edition of his *Lectures*[60] omitted the mathematical and bibliographical part of the first edition, and the *Miscellaneous works* were published too late, in 1855, to be considered a relevant source of information. A few years later, in 1858, William John Macquorn Rankine published his *Manual of Applied Mechanics* which marked the end of the French dominion in the theory of structures[61].

Audoy, 1820; Navier, 1826; Michon, 1857

Following the chronological sequence, the next contributions to the study of the statics of the plate-bande occurred in France. The French military engineer J. V. Audoy published a long memoir dedicated to the application of Coulomb's approach to the calculation of the stability of vaults[62]. Audoy considered mainly the collapse by hinge formation (sliding is impossible) and the problem was to find the position of the «joint of rupture». In normal arches, this leads to complicated algebraic formulae involving the positions of the centers of gravity. Audoy resolved several cases and provided the corresponding expressions, locating the joint of rupture. However, in the case of the plate-bande it was evident that it occurred at the point of intersection of the intrados with the inner face of the buttress. The objective was, of course, to calculate the depth of the buttress and Audoy made the calculations for two plate-bandes of the same span and different thicknesses. For a span of 8 m, a buttress height of 4 m, and thicknesses of 1 and 1,5 m, the depths are 2,85 and 2,45. Surprisingly, the thicker the plate-bande, the more slender is the buttress; but Audoy did not comment on that. Subsequently, Audoy mentioned the possibility of a sliding of the thrust occurring by the sliding of the two half plate-bandes on the springer joints. He concluded that the thrust due to sliding will be always greater than that due to overturning.

Navier, in his *Resumé des leçons*[63] explained at length how to apply Coulomb's approach to any vault. This involves calculating, for every joint, the maxima or minima corresponding to the four ways of collapse. He discussed, first, in an abstract way, the «conditions générales de l'équilibre d'un assemblage de voussoirs» (fig. 24a, left). For each joint, four

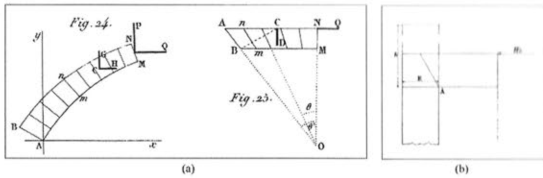
[59] George Rennie made some remarks on the theory of arches, in the discussion following the description of the Wellington bridge. From the text it is evident that he was still within the frame of mind of the old equilibration theory, and unable to understand the importance of Young's contributions, both in arch theory and strength of materials. See G. Rennie's «Discussion» in J. Timperley, «Account of the building of "Wellington" bridge, over the river Aire, at Leeds», in *Minutes and Proceedings of the Institution of Civil Engineers*, vol. III, 1844, pp. 104-114. The discussion was reprinted the same year in America (G. Rennie, «Remarks on the Theory of Arches, made before the Institution of Civil Engineers, London», in *Journal of the Franklin Institute*, vol. VIII, 1844, pp. 226-230).

[60] T. Young, *A Course of Lectures on Natural Philosophy and the Mechanical Arts... A New Edition, with references and notes by the Rev. P. Kelland*, London, Taylor and Walton, 1845.

[61] W. J. M. Rankine, *A Manual of Applied Mechanics*, London, C. Griffin, 1858.

[62] J. V. Audoy, «Mémoire sur la poussée des voûtes en berceau», in *Mémorial de l'Officier du Génie*, 1820, nº 4, pp. 1-96, pl. I-VI.

[63] L. M. H. Navier, *Resumé des Leçons donnés à l'Ecole des Ponts et Chaussés sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines*, Paris, Firmin Didot, 1826.



24 (a) Stability of a system of voussoirs applied to the plate-bande (L. M. H. Navier, *Resumé des Leçons donnés à l'Ecole des Ponts et Chaussés sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines*, Paris, Firmin Didot, 1826, vol. I, pl. I-II). (b) Weight to stabilize the failure by sliding at the head of the buttress (F. Michon, I*nstruction sur la stabilité des voûtes et des murs de revêtement*, Metz, Lithographie de l'École de Metz, 1857).

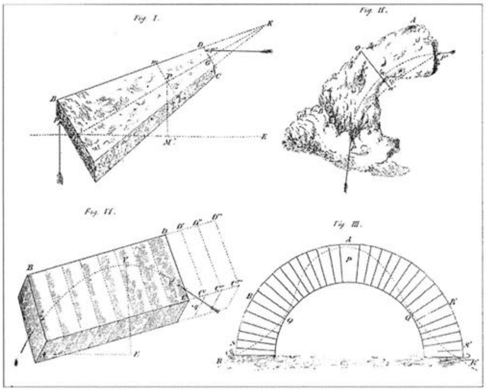
inequalities should be written and the different values of the thrust compared. Of course, this would make the method impracticable, due to the long calculations needed to establish the safety of a simple vault. He took as an example of application a plate-bande in absence of friction or cohesion of mortars, and arrived at the same conclusions as former authors: the joints must converge in one point and the vertical passing through the centre of gravity of the half plate-bande must cut the normal to the springer joint within the masonry. As Giuseppe Venturoli, he wrote the corresponding third-degree equation to obtain the maximum angle for a given span and thickness. The exposition is highly abstract, but, he then arrived at a very simple formula: for slender plate-bandes, the thrust is independent of the thickness and is proportional to the specific gravity γ of the material and the square of the semi-span a (=$s/2$): $H=\frac{1}{2}\gamma\times a^2=\frac{1}{2}\gamma\times s^2$

This formula is equivalent to that of Boistard, discussed above, and due to its simplicity found its way through the manuals of construction[64]. The expression explains the previous results by Audoy (fig. 23): the horizontal thrust is constant for a given span and material, but the weight, which stabilizes the buttress, grows with the thickness. Navier, as Audoy, studied the problem of the thrust by sliding, but these considerations only had a purely theoretical interest. Navier chose the plate-bande as a simple example, but it was not a usual arch for the practice of engineering; the development of arch and vault theory in France during the 19th century corresponded to the engineers and the plate-bande was ignored in the numerous articles on the «stabilité des voûtes» published in France in the next decades[65]. As an exception, F. Michon in 1843 in his «Appendice à la stabilité des voûtes» to his *Instruction sur la stabilité des voûtes*[66] provided a complete analytical study of the plate-bande, following Navier's approach. To simplify the algebra, he considered vertical joints in order to calculate the weights and centers of gravity; the angle of the joint was taken into account when verifying the possibility of sliding. Then, he arrived at the same expression as Navier for the horizontal thrust when collapse occurs by rotation. Michon was aware that the true risk of failure by sliding was not in the plate-bande joints but at the head of the buttress: according figure 24b, the horizontal thrust will tend to make the upper buttress joint at A slide. He then calculated the height h (of the buttress of depth E) above the joint A to avoid the sliding failure. This is in fact the most critical joint in a plate-bande supported by buttresses (of course, if the plate-bande is built within a wall there is no possibility for this type of collapse).

[64] See, for example, J. M. Sganzin, *Programme ou resumé des leçons d'un cours de constructions*, Paris, Carlilian Goeury, 1839, tome I, p. 132.

[65] The contributions were addressed to simplify Coulomb's approach and, in many cases, contained numerical tables of application. See, for example, Petit, «Mémoire sur le calcul des voûtes circulaires», in *Mémorial de l'Officier du Génie*, 1835, n° 12, pp. 73-150, and Garidel, «Mémoire sur le calcul des voûtes en berçeau», *ibid.*, 1835, pp. 7-72.

[66] F. Michon, *Instruction sur la stabilité des voûtes et des murs de revêtement*, Metz, Lithographie de l'École de Metz, 1843.



25 Drawings of the first memoir of Moseley on arch theory (H. Moseley, « On the equilibrium of the arch (Read Dec. 9, 1833) », in *Cambridge Philosophical Transactions*, vol. V, 1835, pl. IX).

Moseley, 1833, 1837

The next contributions to plate-bande theory were made by Henry Moseley who contributed extensively to arch theory from the early 1830's to the mid 1840's. His main contribution was the mathematical definition and study of the concept of line of thrust. Moseley ignored the previous contributions of Young and, apparently, arrived at the idea by his own work[67].

Moseley published a first paper in 1835 (read in 1833 before the Cambridge Philosophical Society) on the theory of the arch. In it, he introduced the concept of « line of pressure » as the line tangent to the successive resultants acting on a family of joints cut in a solid mass (« fig. II » in figure 25). Moseley wrote the equations in a completely general way for the case of parallel plane joints acting on a prismatic body (« fig. I ») in space of three dimensions, and applied them to the case of an inclined plate-bande (« fig. VI »). He arrived at the false conclusion that the line is a parabola.

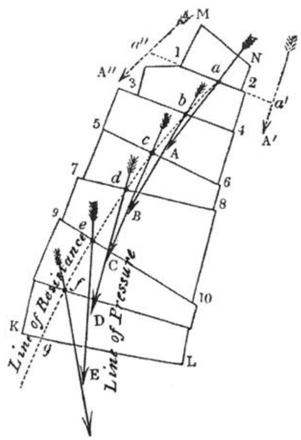
Moseley committed the error of considering the resultant force, or thrust, in each joint as tangent to the line of pressure, i.e., that the point of application of the resultant was the intersection of the plane of joint with the line of pressure. In fact, this only occurs in particular cases. Moseley discovered his error and published his complete, and correct, theory of line of thrust in a second paper in 1838 (read in 1837)[68]. In it, he added a new line, the « line of resistance », which is « the locus of intersections of the consecutive resultants, with the corresponding imaginary surfaces of division »[69]. The concept of the two lines of pressure and resistance was explained more clearly in his treatise *The Mechanical Principles of Engineering and Architecture*, published in 1843[70]. In figure 26, extracted from it, the two lines are represented, and the fact that the thrust need not be tangent to the line of thrust is graphically expressed.

[67] Between 1800 and 1840 several authors arrived to the concept from different points of departure. Thus, the concept was also defined by J. Gertsner, who arrived at it by studying the equilibrium of a system of articulated bars (see J. Gertsner, *Handbuch der Mechanik*, vol. I, Prag, Spurny, 1831). In France, E. Méry, « Mémoire sur l'equilibre des voûtes en berceau », in *Annales des Ponts et Chaussées*, 1840, pp. 50-70, pl. CXXXIII-CXXXIV. Méry took a more practical and engineering approach (however, both Gerstner and Méry make only a passing reference to plate-bandes). Karl-Eugen Kurrer has studied the evolution of the concept of line of thrust in several publications ; for a summary, see K.-E. Kurrer, *The history of the theory of structures. From arch analysis to computational mechanics*, Berlin, Ernst und Sohn, 2008, pp. 213-219. For the implications of the concept in arch analysis, see J. Heyman, *The masonry arch*, Chichester, Horwood, 1982.

[68] H. Moseley, « On the theory of the equilibrium of a system of bodies in contact », in *Cambridge Philosophical Transactions*, vol. VI, 1838, pp. 463-491, with 2 plates.

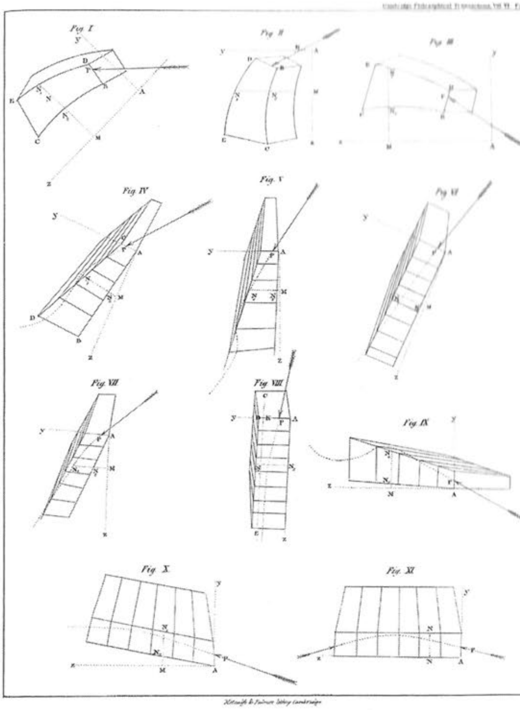
[69] *Ibid.*, p. 464.

[70] H. Moseley, *The Mechanical Principles of Engineering and Architecture*, London, Longman, Brown, Green and Longmans, 1843.



26 Definitions of the «line of resistance» (line of thrust) and «line of pressure». Note that the line of resistance is not tangent to the line of pressure, i.e., the thrust is not tangent to the line of thrust (H. Moseley, *The Mechanical Principles of Engineering and Architecture*, London, Longman, Brown, Green and Longmans, 1843, p. 403).

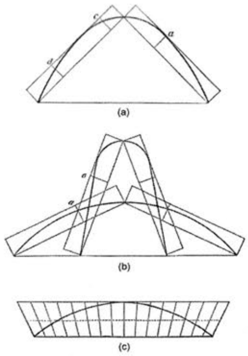
27 Equilibrium of a system of blocks separated by plane joints, as represented by the lines of thrust (H. Moseley, «On the theory of the equilibrium of a system of bodies in contact», in *Cambridge Philosophical Transactions*, vol. VI, 1838, pl. I).



The paper of 1838 (read in 1837) is, like the first, highly mathematical. Moseley wrote the general equations and studied their application to several cases. The drawings in perspective of the first plate, reproduced in figure 27, clearly represent his objective of studying the equilibrium of a system of blocks in the space. In «fig. I» of figure 27, he displayed the general case: the thrust applied to a solid in the space in a general joint. In «fig. II», the joints are horizontal and in «fig. III», vertical. Then, in «fig. IV», he considered a trapezoidal mass with horizontal joints. In each case, he provided the algebraic equation of the line of thrust (line of resistance) in the *yz* plane. Therefore, he was implicitly considering the mass as symmetrical with respect the to *yz* plane, i.e., he was reducing the three-dimensional problem to two dimensions, and obtaining a general expression $y=y(z)$. In the last case of a trapezoidal mass he concluded that the curvature of the line of thrust has a point of inflexion.

Moseley, then, particularized these equations to the case of the buttress («fig. V»). Subsequently, he studied the pier, first as an inclined buttress of uniform section with parallel inclined joints, «fig. VI», or inclined with horizontal joints, «fig. VII», or vertical with horizontal joints, «fig. VIII»[71]. Next, he discussed the general plate-bande, a trapezoidal mass with vertical joints, «fig. IX», the inclined uniform plate-bande, «fig. X», and the horizontal plate-bande «fig. XI». This last case corresponds to the typical plate-bande, and Moseley, demonstrated rightly that the line of resistance is parabolic. Afterwards, like Navier, he deduced that the horizontal thrust is independent of the thickness and is proportional to the square of the span and the specific weight of the stone.

[71] For the vertical pier of uniform section he remarked that a pier of infinite height subject to a finite thrust at the top, would have a finite base. As we have seen, this fact was discovered by Danyzy a hundred years before, in 1732, though it is usually attributed to Moseley.



28 (a–b) Models made by Barlow to demonstrate the practical existence of the line of thrust. (c) Line of thrust in a plate-bande with converging joints (W. H. Barlow, «On the Existence (practically) of the line of equal Horizontal Thrust in Arches, and the mode of determining it by Geometrical Construction», in *Minutes and Proceedings of the Institution of Civil Engineers*, vol. V, 1846, pl. I).

Eventually, Moseley published two more contributions to the theory of arches. However, though important in the context of the history of arches, they added nothing to his previous contributions to plate-bande analysis[72].

Barlow, 1846

The line of thrust approach was not accepted immediately. In 1846, William Henry Barlow considered it necessary to perform some tests with model arches to demonstrate the «Existence (practically) of the line of equal Horizontal Thrust in Arches», i.e., the existence of the line of thrust[73]. To do this, he devised a series of models, and one of them is precisely the triangular arch investigated by Young forty years earlier. Barlow considered first the limit arch formed by two inclined plate-bandes forming 45° with the horizontal (fig. 28a). On the right hand side, the plate-bande has only one joint at *a*, the joint of rupture, corresponding to the limit thickness which just contains the line of thrust[74]; in this case, the arch is in unstable equilibrium and will collapse. However, on the left hand side, the plate-bande has two joints at *c* and *d*, but is stable since the line does not touch the limit of these joints.

In figure 28b, Barlow shows how the unstable arch with joints at *a* is stable for other angles of inclination, as the line of thrust cuts the joints at some distance from the border. Eventually, he draws the line of thrust in a horizontal plate-bande of converging joints. The line drawn is that of minimal thrust. The dotted line is what Barlow calls «the line of impression», which will represent the line of the highest thrust for the plate-bande, passing through the center of gravity of the joints.

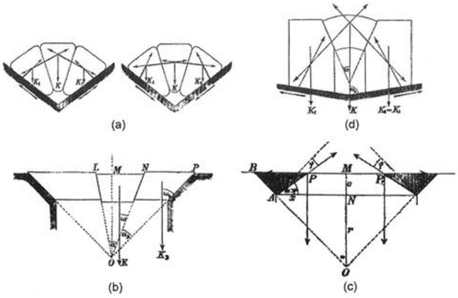
The article by Barlow presents other models of arches, which may be used today in teaching or learning of masonry arch theory[75].

[72] The first of them was published in subscription form in 1839, in the first volume of a collection of essays on bridges (H. Moseley, «On the Theory of the Arch», in *The Theory, Practice and Architecture of Bridges*, London, J. Weale, 1839, vol. I, pp. 1-72, pl. CI-CIII), and then in 1843 when the whole set was published (Id., «The Theory of the Stability of Structures», Part IV in *The Mechanical Principles of Engineering and Architecture*, London, Longman, Brown, Green and Longmans, 1843, pp. 403-485. On plate-bandes, see pp. 429-432).

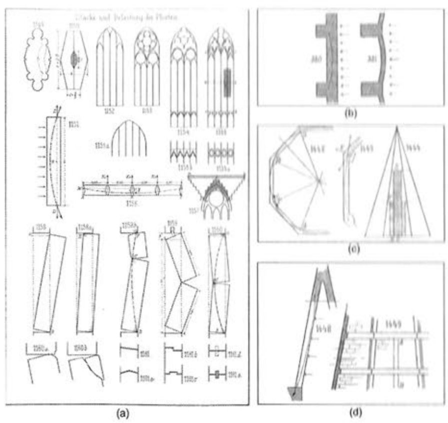
[73] W. H. Barlow, «On the Existence (practically) of the line of equal Horizontal Thrust in Arches, and the mode of determining it by Geometrical Construction», in *Minutes and Proceedings of the Institution of Civil Engineers*, vol. V, 1846, pp. 162-182. Barlow also gives a geometrical method to draw the line of thrust and presents several cases of arches of limited thickness. He treats, also, the design of buttresses. The discussion, in which participated Brunel, Stephenson, Snell, among others, is quite interesting to gauge the state of the art of the English engineers' knowledge of arch theory.

[74] For the limit thickness, Barlow gives 0,1464 the length of the plate-bande, and the joint *a* is located at 0,3535 from the top. These values are correct, and must have been obtained analytically.

[75] The same kind of block models may be made of cardboard, with a technique much easier to use. See S. Huerta, «The use of simple models in the teaching of the essentials of masonry arch behaviour», in G. Mochi, ed., *Theory and practice of constructions : knowledge, means and models*, conference proccedings, Ravenna, 27–29 October, Ravenna, Edizioni Moderna, 2005, vol. II, pp. 747–761.



29 Several problems of failure by sliding or rotation. (a) Three wedges; collapse by falling or rising of the central wedge. (b–c) Study of the plate-bande (cf. with fig. 8 above). (d) Collapse by sliding of the buttresses (A. Ritter, *Lehrbuch der technischen Mechanik*, Hannover, Rümpler, 1865, fig. 275, 276, 278, 283, 284).



30 The plate-bande as a structural element in gothic architecture. (a) Strength of the mullions on gothic windows. (b) Transmission of the horizontal thrust to counter-forts. (c–d) Horizontal and inclined elements in gothic spires (G.-G. Ungewitter, *Lehrbuch der gotischen Konstruktionen. III Auflage neu bearbaitet von K. Mohrmann*, Leipzig, T.O. Weigel Nachfolger, vol. I-II, 1890-1892).

The second-half of the nineteenth century

By the mid-nineteenth century, the equilibrium of plate-bandes was well known. The matter did not appear in the treatises of engineering, but it continued to be described in architectural and construction manuals. The computation of the thrust was not a problem, and to draw a line of thrust using graphic statics was elementary. However, there were a few exceptions, of which we will comment briefly on three. The first one is within the field of theoretical mechanics ; the second is in the context of understanding and restoring some elements of the gothic structure ; and the third is on the practical design of thin shallow arches and domes.

In 1865 August Ritter included in his handbook of applied mechanics, *Lehrbuch der technischen Mechanik*[76], what was, possibly, the last detailed study of plate-bandes. The purpose was not to help architects to design flat arches ; it was an exercise on theoretical mechanics. Ritter studied both the thrust by rotation and by sliding, following Coulomb's approach. Then, he considered, also, the failure by sliding of the buttresses. There were exercises for his lectures on Applied Mechanics delivered at the University of Hannover. Ritter was interested in the problems of friction and his discussion is still useful today when studying arches of plate-bandes with sliding problems. Some of the problems investigated have been reproduced in figure 29.

A generation later, Karl Mohrmann, also in Hannover, played a key role in the application of graphical statics to the study of the gothic structure. His additions to the *Lehrbuch der gostischen Konstruktionen* [Manual of Gothic Construction] by Ungewitter[77] constitute the best and most useful discussions on gothic architecture to date. The reason is his conscientious application of an equilibrium approach through the methods of graphic statics, looking not for the « actual » or « true » state of the structure, but for reasonable equilibrium states which respect the essential characteristic of a (no-tension) masonry material. As we shall see, this approach has been fully validated by modern structural theory.

[76] A. Ritter, *Lehrbuch der technischen Mechanik*, Hannover, Rümpler, 1865. The book had eight editions (1900[8]) and became a standard manual in central and northern Europe.

[77] G.-G. Ungewitter, *Lehrbuch der gotischen Konstruktionen. III Auflage neu bearbaitet von K. Mohrmann*, Leipzig, T. O. Weigel Nachfolger, vol. I-II, 1890-1892.

In gothic architecture it is difficult to find plate-bandes, but Mohrmann saw that the structural behavior of the usual horizontal plate-bandes, and the simple formula deduced for uniform load, could be readily applied to one important element in gothic architecture : the slender vertical posts of gothic windows and the inclined tracery of gothic spires (fig. 30a). He also used the idea of horizontal arches to understand the transmission of the continuous thrust of barrel vaults to the counter-forts (fig. 30b, 30c).

The influence of the plate-bande in the practical calculation of flat arches and domes is evident in the work of Rafael Guastavino. This Spanish master-builder emigrated to USA in the 1880's and exported the traditional Spanish tile vault (*bóvedas tabicadas*) construction. He was not a theorist, though he wrote a book precisely on the theory of tile vaults. He needed to draw the attention and respect of the academic community. His theory is mostly wrong, but in the practical work he used two formulae which are sufficienlty approximate. For arches, the thrust is the thrust of a plate-bande of the same weight and load, i.e., the thrust of a parabolic arch. For domes, he assumed that the thrust is half the thrust of the parabolic arch of the same span and height. One last formula was the one to compute the tension on the iron ring to resist the outward thrust of the dome. With a mastery of the practice and these three formulae, he built during his life thousands of vaults and domes, for some of the most important American buildings. A curious consequence of the theoretic research of the first quarter of the 19th century[78].

The modern theory of masonry structures. Heyman 1966

In the first half of the 20th century, stone plate-bandes continued to be built for masonry buildings, mainly for doors and windows. In many cases, they were not true plate-bandes, but a stone decoration hiding iron beams ; wrought iron and steel were in those years mixed with stone with great freedom[79]. Besides, masonry arches and vaults were seen as elements of the old architecture, and there was a non-written law for the new architecture, « it is forbidden to build arches »[80]. Of course, arches and vaults continued to be built for conservative institutions (the Catholic Church, the Army), and some old professors continued to teach arch and vault construction, until well into the 1950's[81]. But those designs were not published in the Journals and this architecture, which we today may look at with sympathy and interest, was considered anachronic and retrograde.

Masonry arch and vault theory suffered even a greater neglect, and after 1900 it is quite difficult to find original contributions on masonry arches in specialized journals or even in engineering manuals. The theory of structures was written for wrought iron and steel, i.e., for continuous elastic materials[82].

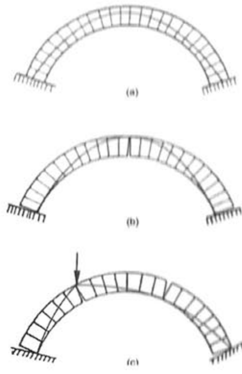
78 For the work of Guastavino, the best contribution is still G. R. Collins, « The Transfer of Thin Masonry Vaulting from Spain to America », in *Journal of the Society of Architectural Historians*, vol. XXVII, 1968, pp. 176-201. See also, S. Huerta, *Las bóvedas de Guastavino en América*, Madrid, Instituto Juan de Herrera, Centro de Estudios Historicos de Obras Públicas y Urbanismo, 2001. For his methods of calculation, see S. Huerta, « The mechanics of timbrel vaults: a historical outline », in A. Becchi, M. Corradi, F. Foce, O. Pedemonte, ed., *Essays in the History of Mechanics*, Basel, Birkhäuser, 2003, pp. 89-133. A new monography will be published shortly : J. Ochsendorf, *Guastavino Vaulting. The Art of Structural Tile*, Princeton, Princeton Architectural Press (forthcoming).

79 See, for example, E. G. Warland, *Modern Practical Masonry*, London, B. T. Batsford, 1929.

80 I owe this ingenious remark to professor Salvador Tarragó, so evident, that i usually goes unnoticed.

81 The persistance of the old teaching in Spanish universities is surprising. Still in 1947, traditional vault construction was one of the main topics in the Syllabus of building construction. See S. Huerta, « Construction History in Spain : Some notes on its current state, historical origins and future perspectives », in A. Becchi, M. Corradi, F. Foce, O. Pedemonte, ed., *Construction History. Research perspectives in Europe*, Florence, Kim Williams Books, 2004, pp. 43-59.

82 A simple perusal of the index of engineering journals and manuals, around 1900, is quite revealing. There are almost no new contributions to masonry arch theory. Only the elastic analysis of masonry arches is considered and the theory was addressed to simplify the heavy calculations involved. See A. Hertwig, « Die Entwicklung der Statik der Baukonstruktionen im 19. Jahrhundert », in *Technikgeschichte*, vol. XXX, 1941, pp. 82-98.



31 Masonry arch. (a) On fixed abutments. One possible line of thrust has been drawn. (b) The abutments have yielded; three cracks open and the position of the line of thrust is determined. (c) A point load is applied either to the arch 31a or 31b. The figure shows the pattern of cracks at collapse (J. Heyman, «The Stone Skeleton», in *International Journal of Solids and Structures*, vol. II, 1966).

However, there were some situations in which certain calculations had to be made: in the analysis and consolidation of heavy damaged buildings, or to ascertain the strength of medieval bridges for the new, much greater, traffic loads. In the field of architecture, the «old» graphic equilibrium approach was used; in fact, there was no other way to attack the problem. The manuals by Ungewitter and Mohrmann, in Germany[83], and by Planat[84], in France, were still used[85]. In the case of the assessment bridges, a majority of engineers felt that elastic analysis should be employed. However, old bridges were visibly cracked and it was not clear how to apply the conventional elastic theory to them. Then, a process of re-discovery began: first, the repetition of tests on voussoir arches, then, a combined method, placing some hinges to simulate cracks and, finally, carrying out an elastic analysis[86].

A new approach was needed to understand the masonry structures. Professor Heyman of Cambridge had the idea, in the 1960's, to apply the modern Limit Analysis (or Plastic Theory) to masonry structures. In his seminal paper «The Stone Skeleton», published in 1966[87], Heyman showed that the Fundamental theorems of Limit Analysis can be translated to masonry provided that the masonry material fulfils three conditions: infinite compressive strength, zero tensile strength and an impossibility of failure by sliding. In a material like this, collapse occurs by the formation of sufficient hinges which convert the structure in a mechanism (indeed, the three conditions assured that a hinge would form when the thrust approximated the boundaries of the masonry). These conditions were more or less explicitly accepted in the «old» arch theory, which we have discussed with reference to the plate-bande.

Within an arch of sufficient thickness, infinite lines of thrust may be drawn, which correspond to infinite inverted catenaries (fig. 31a). If the abutments yield, the arch must crack, thus forming three «hinges» and the line of thrust, passing through them becomes unique (fig. 31b). Therefore, cracks are not dangerous; on the contrary, they are the source of the «plasticity» of masonry. Limit Analysis makes it possible to understand and interpret the different patterns of cracks which are usually due to small movements of the abutments.

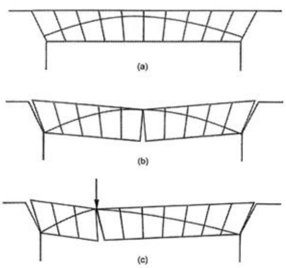
[83] Ungewitter, *op. cit.*

[84] Pierre Planat, editor of the journal *La construction moderne*, published a series of books at the end of the 19th and the beginning of the 20th centuries in which he applied graphic statics to study the equilibrium of arches, vaults and buildings. His books had a great diffusion in France and Spain. See, for example, P. Planat, *L'Art de Bâtir*, Paris, Librairie de la Construction Moderne, vol. III, 1921.

[85] See S. Huerta, «The Analysis of Masonry Architecture: A Historical Approach», in *Architectural Science Review*, vol. LI, 2008, pp. 297-328.

[86] A detailed discussion, with a historical sketch of the development of the assessment methods, in Heyman, *The Masonry Arch*...cit.

[87] J. Heyman, «The Stone Skeleton», in *International Journal of Solids and Structures*, vol. II, 1966, pp. 249-279.



32 Lines of thrust in a plate-bande. (a) Fixed abutments; there are infinite lines of thrust within the masonry. (b) Small yielding of the abutments; the position of the line is determined by the cracks. (c) Effect of a point load in the cracked plate-bande (drawings by the author).

Eventually, a point load is applied on the same arch, either in the uncracked situation of figure 31a or in the cracked one of figure 31b. The action of the load modify the form of the line of thrust, and when it becomes so distorted as to be contained just within the masonry, four hinges form and this leads to a four-bar mechanism of collapse. This final state is independent of the original state of the arch. In the case of figure 31b during the process of load, some cracks will open and other close following a process that is heavily dependant on the state of the joints or new yielding of the abutments. Nevertheless, the final collapse load is unique. Moreover, the strength of the arch is independent of the strength of the individual stones. The collapse occurs without reaching the crushing strength; it is a matter of stability.

Within this theory, some fundamental theorems have been demonstrated. In particular, the *Safe Theorem* states that if it is possible to find a distribution of internal forces in equilibrium with the external loads and which does not violate the yield condition, the structure is safe, i.e., it will not collapse. A line of thrust represents a set of internal forces in equilibrium with the loads; the yield condition is that the material must work in compression. If it is possible to draw a line of thrust within the arch, the arch will not collapse. Therefore, the arch in figure 31a is perfectly stable, and no matter what small movements of the abutments may occur or may lead to different crack patterns, the arch will stand.

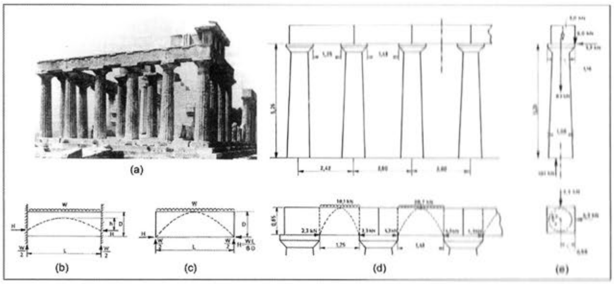
As Heyman remarked this validates the «equilibrium approach» for the analysis of masonry structures (in fact, for any structure built with a ductile material which behaves in a ductile way under a load). The equilibrium approach is at the heart of the «old» theory of masonry arches. We are now in a paradoxical situation: the «old» theory is the «modern» theory, within the framework of Limit Analysis of the 20th century, and the elastic analysis, nowadays performed by computer with FEM packages, pertains to the old 19th century approach[88].

Heyman, 1972. Trabeated architecture

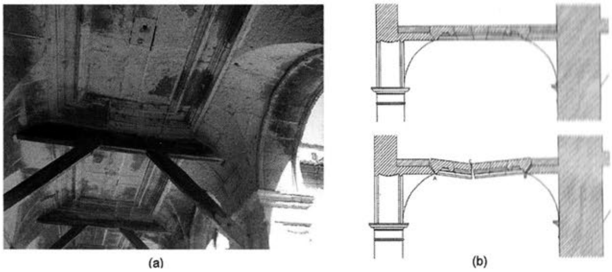
A new insight in the theory of plate-bandes was supplied by Heyman, within the framework of Limit Analysis. In the provocative article «Gothic construction in ancient Greece»[89] he demonstrated the equivalence between monolithic masonry lintels and plate-bandes. A lintel may break – indeed many lintels are broken – and there is therefore no distinction between lintels and plate-bandes. A well designed lintel should have the proportions and boundary conditions to work as a plate-bande, exercising a horizontal thrust. In fact, the modern theory of masonry structures throws new light on the structural behavior of the trabeated architecture, formed by columns and lintels, of the Egyptians and Greeks. The plate-bande contained between fixed abutments functions like an arch and there are

[88] For a concise and illuminating discussion of the actual situation of the theory of structures, see J. Heyman, *Basic Structural Theory*, Cambridge, Cambridge University Press, 2008.

[89] J. Heyman, «Gothic Construction in Ancient Greece», in *Journal of the Society of Architectural Historians*, vol. XXXI, 1972, pp. 3-9.



33 Static analysis of the Temple of Afaia Egina. (a) External view and general dimensions. (b–c) Equilibrium of an uncracked and cracked lintel. (d) Forces at the top of the columns. (e) Equilibrium of the corner column (J. Heyman, « Gothic Construction in Ancient Greece », in *Journal of the Society of Architectural Historians*. vol. XXXI, 1972, pp. 3–9).



34 Flat vault of the cloister in the old Franciscan convent of Llucmajor, Mallorca. (a) View of the shoring put a century ago, and actual state (below) after its removal. (b) Explanation of the cracks : original and cracked (below), safe, situation of the flat vault (S. Huerta, E. Rabasa, *Informe sobre la estabilidad y consolidación de las bóvedas del claustro de Sant Bonaventura de Llucmajor (Mallorca)*, Madrid, Technical report of the Polytechnic University, 2006).

infinite lines of thrust in equilibrium with the weights of the stones. One of them has been drawn in figure 32a. If the abutments yield slightly, three cracks will form and the position of the line of thrust is unique and corresponds to the minimum thrust (fig. 32b). The application of a point load will have no effect on a plate-bande on fixed abutments : there is no disposition of hinges giving rise to a mechanism of collapse, and the load can grow until the crushing of the stones. The effect of the point of load in the cracked plate-bande of figure 32b will simply be to change the position of the upper crack. But, again, there is no mechanism of collapse. In fact, the only way of collapse is by « snap-through » due to the overturning of the abutments (cf. « fig. 5 » in figure 6, above).

In the case of a Greek temple, the lintels may be imagined as broken. In his paper, Heyman expressed with outmost clarity the new radical view of trabeated architecture : « [...] it would be wise, in analyzing the stability of a Greek trabeated structure, to assume that all the masonry is cracked. If a complete and satisfactory analysis can be made for the cracked structure, then this is a complete proof that the original structure, cracked or not, is also stable »[90].

A new vision emerges. For example, the proportion of the columns of a Greek temple does prevents the failure by overturning due to the occasional fracture of a lintel, in particular during construction (or now in the incomplete ruined state of many temples). Figure 33 summarizes the static analysis of the Temple of Afaia Egina made by Heyman. If the corner lintel breaks, a thrust will try to overturn the external column, which is acting as a buttress.

The apparent simplicity of the process hides a complex theory. As always with good theories, the conclusions are simple. However, the right application of these ideas requires a mastery of the theory and practical experience. In figure 34a, a masonry flat vault has been shored. These wooden supports have been in place for one hundred years, and now need to be removed. This is not an academic question; the architect or engineer who removes the shores is accepting the responsibility of whatever happens afterwards. Figure 34b explains

[90] *Ibid.*, p. 6.

the problem, a small yield of the walls, and the way to attack the analysis. Today, the shores have been removed and the distorted flat vaults stand safely (superior stell ties had to be added to secure the buttress systems against the new loads)[91].

Heyman, 1995, 2005. Spires and rose-windows

Plate-bandes are present in two other complex gothic structural types: the spire and the great windows and rose-windows. In fact, they are not «actually» present, but the analyst may consider that both spires and rose-windows function as combinations of plate-bandes, and this consideration may help the analyst to find a reasonable equilibrium state, which, also agrees with the movements of the fabric expressed in cracks and leanings.

Gothic spires are, invariably, hollow pyramids with an octagonal base. They are light structures and the action of wind is crucial. There are two structural parameters: the angle of the faces (or of the edges) and the thickness of the wall. The wall, in turn, may be solid or formed of a tracery of ribs. Heyman studied this problem in different papers and books[92]. The overall stability gives a value of the thickness. For a dead load, a membrane solution of an analogue cone leads to a simple geometrical rule: the thickness must allow a circle to be inscribed within the thickness (fig. 30c above). As an alternative, a skeleton of ribs may be imagined to transmit the loads within the masonry of the spire. These ribs may actually exists, as in tracery spires or be virtual ribs. Eventually, the danger of collapse by snap-through of a panel circumscribed by ribs may be analysed. By studying the equilibrium of an analogue plate-bande, Heyman deduced a geometrical rule agreeing very well with the empirical rules given by Mohrmann[93].

Gothic windows and rose-windows are more complex structures. They consist of very slender stone mullions or tracery ribs[94]. To the pattern of the tracery is superposed an ironwork fixing the glasses and contributing to the stability of the whole. Any intent of finding the «actual» state of internal forces in such a complex and highly hyperstatic structure would be futile. Heyman was the first to make a structural analysis of this kind of structure[95]. Following the equilibrium approach, i.e., the Safe Theorem, Heyman searched for reasonable states of equilibrium. He considered the structure to be composed primarily of the stone elements and the ironwork to only transmit the wind load to the stone skeleton. Since the structure is symmetrical, it is logical to presume a symmetrical distribution, and Heyman assumed a «domical» distribution of forces. In other words the inclined thrust at the end of the radial spokes are tangent to a domical surface inscribed in the thickness of the masonry. This assumption renders it possible to calculate, easily, the total horizontal thrust H^*. For small values of t/d (thickness/diameter), which is the case in rose-windows (usually, 1/20 - 1/40), $H^* = Wd/4t$. By dividing this total thrust by the number of spokes, the individual thrusts are obtained. Then, as the internal

[91] S. Huerta, E. Rabasa, *Informe sobre la estabilidad y consolidación de las bóvedas del claustro de Sant Bonaventura de Llucmajor (Mallorca)*, Madrid, Technical report of the Polytechnic University, 2006 (E-print: www.ad.upm.es).

[92] J. Heyman, «Spires and Fan Vaults», in *International Journal of Solids and Structures*, vol. III, 1967, pp. 243-258; «Hemingbrough Spire», in *Structural Repair and Mantenance of Historical Buildings II*, Southampton, Computational Mechanics Publications, 1991, vol. I, pp. 1-9; «Spires», Chap. 7 in *The Stone Skeleton. Structural Engineering of Masonry Architecture*, Cambridge, Cambridge University Press, 1995, pp. 127-138.

[93] G.-G. Ungewitter, «Steinerne Turmhelme», in *Lehrbuch*...cit., vol. II, pp. 595-614.

[94] An old late-gothic rule assigns to the mullions a thickness between 1/30 and 1/42 of the span of the main nave. See Huerta, *Arcos, bóvedas y cúpulas*...cit., p. 160-162.

[95] J. Heyman, «Rose Windows», in H. R. Drew, S. Pellegrino, ed., *New Approaches to Structural Mechanics, Shells and Biological Structures*, Dordrecht, Kluwer, 2002, pp. 115-125. Published also in Becchi, Corradi, Foce, Pedemonte, ed., *Essays in the History of Mechanics* cit., pp. 165-177. R. Barthel has studied also the structural behavior of gothic windows. See R. Barthel, L. Schiemann, M. Jagfeld, «Static analysis and evaluation of the construction system of a gothic "choir-window" consisting of a filigree tracery and slender stone ribs», in Huerta, ed., *Proceedings of the First International Congress on Construction History*...cit., vol. I, pp. 333-340.

forces form small angles with the plane of the rose-window, there is no need to compose the vertical forces and, in fact, it becomes a problem of resolving forces in the plane. Heyman gives the calculations for the rose of Notre Dame the Mantes, with a diameter of 8 m and spokes of nearly 250 × 250 mm (t/d = 1/32). For a wind of 2 kN/m2, the average compressive stress is around 1 N/mm^2, which is very low in comparison to the crushing strength of a medium stone.

Conclusions

The plate-bande entered into the theory of structures as a particular problem of the more general theory of the masonry arch which began in the last quarter of the 17th century. The plate-bande is a «bad» arch ; if the ideal arch is a curve (Hooke's inverted catenary), the plate-bande is straight. It needs to be «thick» to contain an arch within the stones. As a consequence of its form the plate-bande presents particular problems : it tends to deform downwards forming a visible kink in the intrados, and its thrust is much greater than that of any other arch of the same span. Its use was usually restricted to architecture to form the lintels of windows and gates, without the necessity of employing great monolithic stones. It was sometimes used at the gates of certain fortifications, but this is rare. It has never been used as a main element in civil engineering.

However, as we have seen, this «inconvenient» arch played an important role in the development of the theory of the arch. For La Hire, Bélidor and Navier, it was a good and simple example to explain a complicated theory and provided simple calculations. For Young, it presented a problem which compelled him to adopt a completely new approach to arches : the study of possible solutions of equilibrium within the masonry with the help of the line of thrust (curve of equilibrium) concept. It was the study of the triangular arch formed by two inclined plate-bandes, a very old but most uncommon form, which faced him with the necessity of freeing the internal forces from the strait-jacket of the intrados and the inclination of the joints. For Moseley, the plate-bande was, again, the simple example to be used to verify the highly general algebraic equations of «the equilibrium of a system of bodies in contact» represented by the line of thrust (line of resistance). Eventually, Barlow used triangular arches to prove the «existence in practice» of the line of thrust.

With the advent of graphic statics and the division between architecture and engineering, the simple plate-bande disappeared from engineering manuals. However, previous discoveries with regard to plate-bande behavior find a place in practical architecture and engineering; the thrust of a horizontal plate-bande is, also, the thrust of a parabolic arch. The expression $H = Ws/8h$, where W is the total load, s the span, and h the height of the arch plus its thickness (or the thickness of the plate-bande), was widely used in the last quarter of the 19th century to compute, conveniently and with enough precision, the thrust of surbased and flat arches. Rafael Guastavino built thousands of square meters of flat brick tile vaults using it. The formula was also used by Mohrmann to compute the stability of the vertical mullions of the gothic windows.

Heyman, who built the modern theory of masonry structures, employed the plate-bande to explain the structural behavior of flying-buttresses, Greek temples, gothic spires and rose-windows. Today, the plate-bande continues to present a problem to architects or engineers involved in the consolidation of buildings. Knowledge of the historical development of this modest structural element may be very useful in the double task facing modern technicians : first, «unlearn» the classical elastic theory of frames and trusses, usually taught in the universities (to free oneself from what Heyman calls «Navier's strait-jacket»), and, then, learn the theory in the context of the in-depth study of the existing buildings.

L'architrave le plancher la plate-forme

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sous la direction de Roberto Gargiani

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L'auteur et l'éditeur remercient l'Ecole polytechnique fédérale de Lausanne pour le soutien apporté à la publication de cet ouvrage.

La collection «Architecture Essais» est dirigée par les professeurs Jacques Lucan et Luca Ortelli

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Traduction en français: Marie-Christine Lehmann
(textes de Salvatore Aprea, Maria Chiara Barone, Antonio Becchi, Gianluca Belli, Alberto Bologna, Massimo Bulgarelli, Antonio Burgos Nunez, Vittorio da Caiano, Giulia Chemolli, Giovanni Di Pasquale, Francesco Di Teodoro, Roberto Gargiani, Jean-Pierre Goguet, Albert Guettard, Riccardo Gulli, Tullia Iori, Daniel Jeanneret, Louis Jeanneret, Charles Lafaye, Beatrice Lampariello, Antoine de Maillet, Michele Messeri, Emanuela Montelli, Marc Nollet, Pier Nicola Pagliara, Michael Petzet, Mario Piana, Marco Pogacnik, Sergio Poretti, Francesco Quinterio, Bruno Reichlin, Nicola Rizzi, Anna Rosellini, Giampiero Sanguigni, Hermann Schlimme, Philipp Speiser, Claude Tibières, Valerio Varano, Etienne Vignon et Robin Wimmel).

Recueil de textes édités sous la direction du Laboratoire de Théorie et d'Histoire de l'Architecture 3: Roberto Gargiani, Salvatore Aprea, Maria Chiara Barone, Giulia Chemolli, Beatrice Lampariello, Marie-Christine Lehmann, Khue Tran.

Cet ouvrage est une publication des Presses polytechniques et universitaires romandes, fondation scientifique dont le but est principalement la diffusion des travaux de l'Ecole polytechnique fédérale de Lausanne et d'autres universités francophones.
Le catalogue général peut être obtenu par courrier aux:
Presses polytechniques et universitaires romandes, EPFL-Centre Midi, CP 119, CH-1015 Lausanne, par E-mail à ppur@epfl.ch, par téléphone au (0)21 693 41 40 ou encore par fax au (0)21 693 40 27.
www.ppur.org

Première édition 2012

ISBN 978-2-88074-893-7

Imprimé en Italie